EECS 16A - APS 2

LAST LAB! :)

TA, TA, ASE
Announcements!

- This is the **last lab**!
- Do APS1 first if you haven’t yet (APS2 can then be done during buffer)
- Course evaluations: [link](#)
- APS buffer labs 12/7-12/11 (RRR week)
  - Sign up here: [tinyurl.com/aps-buffer](http://tinyurl.com/aps-buffer)
  - Encouraged to attend a Mon-Wed section
- Good luck on the final!

^ A pre-quarantine meme, a true 16A lab
Last lab: APS 1

- Cross correlated beacons with received signal
- Found the offsets (in samples) between peaks, converted to TDOAs, and calculated distances from each beacon
- **What was the missing piece that we needed to calculate distance?**
  - Hint: we don’t have absolute times of arrival for all the beacons, only relative offsets.
3 beacon example

- Let beacon centers be: \((x_0, y_0), (x_1, y_1)\) and \((x_2, y_2)\)
- Time of arrivals: \(t_0, t_1, t_2\)
- Distance of beacon \(m\) (\(m = 0, 1, 2\)) is \(d_m = vt_m = R_m\) (circle radii)

Circle equations: \((x - x_m)^2 + (y - y_m)^2 = d_m^2\)
Problem: We don’t know $t_0$

- Only know time offsets: $\tau_m = t_m - t_0$
- $R_m = \sqrt{(x - x_m)^2 + (y - y_m)^2} = v_s t_m$
- $R_0 = \sqrt{x^2 + y^2} = v_s t_0$ (Beacon 0 is at origin)
- $R_m - R_0 = v_s (t_m - t_0) = v_s \tau_m$
Setting up n-1 hyperbolic equations

- $m \neq 0$ (as $\tau_0 = 0$)
- This is the equation for a hyperbola
- This is hard to solve

\[ R_m - R_0 = v_s \tau_m \]

\[ v_s \tau_m = \frac{-2x_mx + x_m^2 - 2y_my + y_m^2}{v_s \tau_m} - 2\sqrt{x^2 + y^2} \]
Making it linear:

- Same trick: subtract first equation from others

\[
v_s \tau_m - v_s \tau_1 = \left[ \frac{-2x_m x + x_m^2 - 2y_m y + y_m^2}{v_s \tau_m} \right] - \left[ \frac{-2x_1 x + x_1^2 - 2y_1 y + y_1^2}{v_s \tau_1} \right] - 2\sqrt{x^2 + y^2}
\]

Linear! simplify! \( m \neq 0, m \neq 1 \)
Making it linear:

- After simplifying, we have n-2 linear equations and 2 unknowns (x,y)
- Can do least-squares regardless of number of beacons

- Best estimate of location if measurements are inconsistent
- If there is no exact point of intersection bc of error or noise

\[
\begin{align*}
\frac{2x_m}{v_s \tau_m} - \frac{2x_1}{v_s \tau_1} & \quad (x) \\
\frac{2y_m}{v_s \tau_m} - \frac{2y_1}{v_s \tau_1} & \quad (y) = \left( \frac{x_m^2 + y_m^2}{v_s \tau_m} - \frac{x_1^2 + y_1^2}{v_s \tau_1} \right) - (v_s \tau_m - v_s \tau_1)
\end{align*}
\]

\[
Ax = b
\]

\[
A^T Ax = A^T b
\]
Setup Looks Like:
Important notes

- Read over the math **carefully**, We’ll be asking you about it!
- Stay safe and good luck with the rest of the semester! *Virtual hand wave*
  - Thank you for being part of this remote offering!