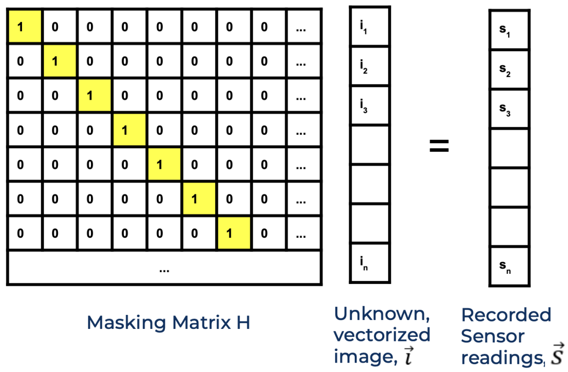
EECS 16A Designing Information Devices and Systems I
Spring 2023 Pre-Lab Reading Imaging 3

1 Imaging 2 Review

Last week, you built your first pixel scanning camera! We used pixel-by-pixel scanning to get a scan of a picture in a cardboard box. The 'golden equation' which represented our system from Imaging 2 was $\vec{s} = H\vec{i}$:



If you recollect, we got our unknown image vector $\vec{i}$ from our mask matrix H and the sensor reading $\vec{s}$ by multiplying by H^{-1} on both sides of the 'golden equation' to get: $\vec{i} = H^{-1}\vec{s}$. To be able to do this, we need our mask matrix to **always** have the following properties:

(a) Invertible

(b) Linearly independent columns

(c) Trivial nullspace

(d) Non-zero determinant

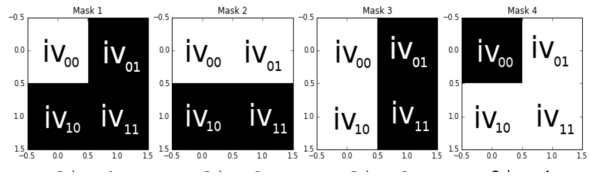
(e) Unique solution to the linear system, $H\vec{i} = \vec{s}$, where $\vec{s}$ is any sensor reading, and $\vec{i}$ is any unknown image.

For square matrices (as we're dealing with), these properties are all equivalent.

2 Imaging 3 Overview

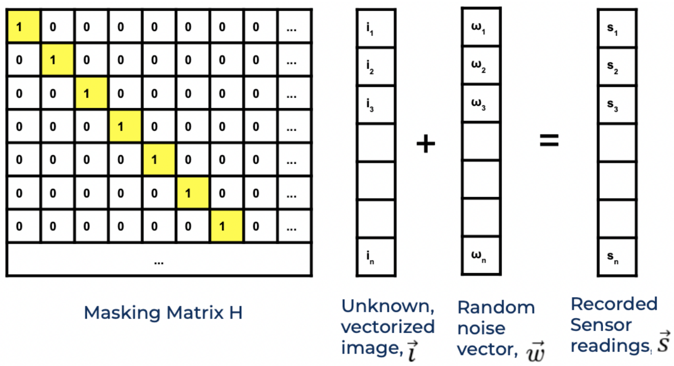
In Imaging 2, we defined what a "good measurement" is. If we want to always reconstruct our image, we need an invertible mask matrix, H. However, is invertibility the only condition for a "good measurement"? Are all invertible matrices good for imaging? What happens if you shake your projector in the middle of a scan? What if we consider scanning multiple pixels at a time instead of just one? These are some questions we will look to answer in Imaging 3! We will use the same matrix/vector representation for $H, \vec{i}$ and $\vec{s}$ as Imaging 2. In Imaging 3, we will look into multi-pixel scanning. Here are some examples of simple 2x2 mask matrices that are used for multi-pixel scans:
As you can tell from the image, we now have multiple 1's in each row of the mask matrix H, which means we are scanning multiple pixels at a time. Instead of each measurement (scan) corresponding to one pixel, measurements are now **linear combinations** of pixels. Note that our system is still the same. We still have our 'golden equation', $\vec{s} = H\vec{i}$, and we still need H to be invertible. To motivate multipixel scanning, we first need to understand noise better.

Noise is any unwanted, random variation in our measurement; for example, additional room light getting into our



projector boxes to further brighten the image is noise. If there is too much noise in the system, it makes it very hard to distinguish between the actual signal and the noise. We ideally want high signal and low noise in our measurements. A measure of a system's perturbation by noise is the signal-to-noise ratio (SNR). A high SNR is extremely desirable in any physical system.

How does noise affect our model?



The only difference is we now have an added noise vector, $\vec{w}$. Our new 'golden equation' now becomes $\vec{s} = H\vec{i} + \vec{w}$.

How does multi-pixel imaging relate to noise? When we scan multiple pixels at a time by illuminating them, it makes the box much brighter when compared to the light in the box from the illumination of a single pixel. We can leverage multi-pixel image by taking the **average** of all our scans. Signal averaging is a very common technique used to increase the SNR. Here's a simple example to illustrate why: Let's say you are scanning a small 4x4 image. During some of your scans, a Stanford student thinks it would be very funny to shine a super bright torchlight into your box. In the single-pixel imaging model, you only scan each pixel once. This means that you only have bad, unusable data for the scans that were disturbed. In the multi-pixel imaging model, you scan each pixel multiple times. Thus, even if some of the scans are bad, averaging across all the scans for every pixel will give us a better final result that just using a singular bad measurement (single-pixel model). Note that averaging is not going to completely solve the problem. It will, however, improve our results.

2.1 Eigenvalues

Eigenvalues??!! What do eigenvalues have to do with Imaging you may ask. Just like in Imaging 1, if you multiply by H^{-1} on both sides of the (new) golden equation, you get $\vec{i_{est}} = H^{-1}\vec{s} = \vec{i} + H^{-1}\vec{w}$. Don't worry if this doesn't make sense right now. We will leave the derivations for lab, so just assume that this is correct for now. You now have an $H^{-1}\vec{w}$ term that you can leverage to control the affect of noise (since H is something that you defined). Remember that you want high SNR which means low noise. Do you think that means you would want an H with a large or small multiplicative effect? The "multiplicative effect" of a matrix can be defined by its eigenvalues.

Eigenvalue Theory: If H is an $N \times N$ matrix that you know is linearly independent (invertible), it has N linearly independent eigenvectors as $Hv_i = \lambda_i v_i$ for $i = 1...N$, where λ_i is the i$^{\text{th}}$ eigenvalue for the i$^{\text{th}}$ eigenvector. An interesting fact about eigenvalues is, if λ is an eigenvalue of H, $\frac{1}{\lambda}$ is the eigenvalue for H^{-1}. This should give you a hint regarding how we can leverage eigenvalues to reduce the effect of the noise term. We will go over the proof of this fact during lab next week!

You may now be inclined to think that multi-pixel imaging is miles ahead of single-pixel imaging. However, for reasons that you will find out in lab, multi-pixel may not always be better than single-pixel imaging. Please don't forget to bring your lab kits as you complete your journey through the Imaging module and become expert photographers, one (or many) pixel(s) at a time!

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