Toolbox

- Resistors
- Capacitors
- Open-circuits
- Voltage Dividers/Summers
- Op-Amps
- Thevenin and Norton Equivalence
- KCL/KVL
- Element Definitions
- DAC
- Negative Feedback
- Op-Amp in Negative Feedback
- “Golden Rules” for Op-Amps

GR #1: \( I_+ = 0, \quad I_- = 0 \) no current into OpAmp

GR #2: in negative feedback: \( U^+ = U^- \)
Today

**Voltage Divider**

\[ V_{R2} = V_S \left( \frac{R_2}{R_1+R_2} \right) \]

**Voltage Summer**

\[ V_{\text{out}} = V_1 \left( \frac{R_1}{R_1+R_2} \right) + V_2 \left( \frac{R_1}{R_1+R_2} \right) \]

**Unity Gain Buffer**

\[ \frac{v_{\text{out}}}{v_{\text{in}}} = 1 \]

**Inverting Amplifier**

\[ v_{\text{out}} = v_{\text{in}} \left( - \frac{R_f}{R_x} \right) + V_{\text{REF}} \left( \frac{R_f}{R_x} + 1 \right) \]

**Non-inverting Amplifier**

\[ v_{\text{out}} = v_{\text{in}} \left( 1 + \frac{R_{\text{top}}}{R_{\text{bottom}}} \right) - V_{\text{REF}} \left( \frac{R_{\text{top}}}{R_{\text{bottom}}} \right) \]

**Transresistance Amplifier**

\[ v_{\text{out}} = i_{\text{in}}(-R) + V_{\text{REF}} \]
Checking for Negative Feedback

Step 1 – Zero out all independent sources
- replacing voltage sources with wires
- current sources with open circuits as in superposition

Step 2 – Wiggle the output and check the loop – to check how the feedback loop responds to a change.
- if the \((U^+ - U^-)\) decreases, the output \(A(U^+ - U^-)\) must also decrease. The circuit is in negative feedback
- if the \((U^+ - U^-)\) increases, the output \(A(U^+ - U^-)\) must also increase. The circuit is in positive feedback
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Element Definitions:

\[ V_{R_1} = I_1 R_1 \]
\[ V_{R_2} = I_1 R_2 \]
\[ V_{R_1} = U_1 - U_2 = V_{in} \]
\[ V_{R_2} = U_2 - U_3 = -V_{out} \]

\[ U_1 = V_{in} \]
\[ U_3 = V_{out} \]
\[ U_2 = 0 \]
\[ U_2 = U^- \Rightarrow \text{NFB} \Rightarrow U^- = U^+ = 0 \]
\[
U_1 = V_{in} \\
U_3 = V_{out} \\
U_2 = 0 \\
U_2 = U^- \Rightarrow \text{NFB} \Rightarrow U^- = U^+ = 0
\]

Element Definitions:
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\end{align*}
\]

(KCL)
\[
I_1 = I_2 + I_-
\]

(GR#1)
\[
U_2 = U^- \Rightarrow \text{NFB} \Rightarrow U^- = U^+ = 0
\]
\[ U_1 = V_{in} \]
\[ U_3 = V_{out} \]
\[ U_2 = 0 \]

\[ U_2 = U^- \Rightarrow \text{NFB} \Rightarrow U^- = U^+ = 0 \]

\[ \text{Element Definitions:} \]
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\[ I_1 = I_2 + I_- \] \hspace{1cm} \text{(GR#1)}

\[ V_{in} = I_1 R_1 \]
\[ V_{out} = -I_2 R_2 \]
\[ I_2 = I_1 \]

\[ -\frac{V_{out}}{R_2} = \frac{V_{in}}{R_1} \]

\[ V_{out} = -\frac{R_2 V_{in}}{R_1} \]

\[ A_v = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1} \]

**Inverting Amplifier!**
A faster way...

NFB ⇒ Golden Rule #2 ⇒ $U^- = U^+$

$U^+ = 0 \Rightarrow U^- = 0 \Rightarrow U_2 = 0$

GR #1 + KCL

$I_1 = I_2 + I_-$

$\frac{U_1 - U_2}{R_1} = \frac{U_2 - U_3}{R_2}$

$\frac{V_{in}}{R_1} = -\frac{V_{out}}{R_2}$

$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$
Example circuit 2 (trans-resistance amplifier)

Zero-out independent sources
Example circuit 2 (trans-resistance amplifier)

Zero-out independent sources

From GR #1:

\[ I_+ = 0 \Rightarrow I_2 = 0 \]
\[ \Rightarrow U_2 = U_1 \]

Check for negative feedback

\[ A \left( U^+ - U^- \right) \]

Increasing output, increases \( U^+ \), increases output

Not in Negative feedback 😭
Example circuit 2 (trans-resistance amplifier)

Invert Polarity!

Zero-out independent sources

From GR #1:

\[ I_\text{in} = 0 \quad \Rightarrow \quad I_2 = 0 \]
\[ \Rightarrow \quad U_2 = U_1 \]

Check for negative feedback

Increasing output, increases \( U^- \), decreases output

in Negative feedback 😍
Example circuit 2 (trans-resistance amplifier)

Invert Polarity!

\[ I_r \]

\[ R \]

\[ V_{\text{out}} \]

\[ U_1 \]

\[ U_2 \]

\[ I_{\text{in}} \]

\[ I_{\text{+}} \]

\[ I_{\text{−}} \]

NFB ⇒ Golden Rule #2 ⇒ \( U^- = U^+ \)

\( U^+ = 0 \) ⇒ \( U^- = 0 \) ⇒ \( U_1 = 0 \)

Golden Rule #1 & KCL

\[ I_{\text{in}} = I_r + I_{\text{+}} \]

\[ I_{\text{in}} = \frac{U_1 - U_2}{R_1} \]

\[ V_{\text{out}} = -R_1 I_{\text{in}} \]

Input current, output is voltage!
Example circuit 3 -

\[ V_{in} + \rightarrow I_+ \rightarrow + \rightarrow I_- \rightarrow - \rightarrow V_f \]

\[ V_f \rightarrow R_1 \rightarrow + \rightarrow V_{in} - \rightarrow + \rightarrow + \rightarrow - \rightarrow - \rightarrow - \rightarrow - \rightarrow - \]
Example circuit 3 -
Example circuit 3 -

零出独立源

\[ V_{in} \quad + \quad I_+ \quad + \quad I_- \quad - \quad V_f \]

Zero-out independent sources
Example circuit 3 -

Zero-out independent sources

Check for negative feedback

\[ A \left( U^+ - U^- \right) \]

Increasing output, decreases \( U^- \), increases output

Not in Negative feedback 😭
Example circuit 3 -

Zero-out independent sources

Check for negative feedback

\[ A \left( U^+ - U^- \right) \]

Increasing output, decreases \( U^+ \), decreases output

in Negative feedback 😍
Example circuit 3 -

NFB $\Rightarrow$ Golden Rule #2 $\Rightarrow U^- = U^+$

$\Rightarrow V_{in} = -V_f$

Voltage divider:

$$V_f = \frac{R_2}{R_1 + R_2} V_{out}$$

$$V_{in} = -\frac{R_2}{R_1 + R_2} V_{out}$$

$$A_v = \frac{V_{out}}{V_{in}} = -\frac{R_1 + R_2}{R_2} = -\left(1 + \frac{R_1}{R_2}\right)$$
Artificial Neuron

- Neurons in our brain are interconnected.
- The output of a single-neuron is dependent on inputs from several other neurons.
- This idea is represented with vector-vector multiplication – the output is a linear combination of several inputs.
- An artificial neuron circuit must perform addition and multiplication.

\[
\begin{bmatrix}
  w_{1j} & w_{2j} & \cdots & w_{nj}
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_n
\end{bmatrix}
= \sum_{i=1}^{n} w_{ij}x_i
\]
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Zero-out independent sources
Artificial Neuron

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![Diagram of an artificial neuron circuit]

- Zero-out independent sources
- Check for negative feedback

\[ A \left( U^+ - U^- \right) \]

Increasing output, increases \( U^- \), decreases output in Negative feedback 😍
Artificial Neuron

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- The output of a single-neuron is dependent on inputs from several other neurons.
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\[ \text{NFB} \Rightarrow \text{Golden Rule \#2} \Rightarrow U^- = U^+ \]
\[ U^+ = 0 \Rightarrow U^- = 0 \]

KCL:
\[ I_1 + I_2 = I_3 + I_\text{out} \]

\[ \frac{U^- - V_1}{R_1} + \frac{U^- - V_2}{R_2} = \frac{V_{out} - U^-}{R_3} \]

\[ -\frac{V_1}{R_1} - \frac{V_2}{R_2} = \frac{V_{out}}{R_3} \]

\[ V_{out} = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2 \ldots - \frac{R_3}{R_i}V_1 \ldots \]
Artificial Neuron

\[ V_{out} = - \frac{R_3}{R_1} V_1 - \frac{R_3}{R_2} V_2 \ldots - \frac{R_3}{R_i} V_i \ldots \]

Q: All weights are negative. How can we change sign?
A: Add an inverting amp circuit?
Artificial Neuron

\[ V_{out} = -\frac{R_3}{R_1} V_1 - \frac{R_3}{R_2} V_2 \cdots - \frac{R_3}{R_i} V_i \cdots \]

Q: All weights are negative. How can we change sign?

Q: Can we inverting amp circuit?
Artificial Neuron

\[ V_{\text{out}} = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2 \ldots - \frac{R_3}{R_i}V_i \ldots \]

Q: All weights are negative. How can we change sign?

Q: Can we add an inverting amp circuit?

A: Not always…. But perhaps here is OK.

Q: What's the requirement on Rth?

A: $Rth = 0$?
Artificial Neuron

\[ V_{out} = -\frac{R_3}{R_1} V_1 - \frac{R_3}{R_2} V_2 \ldots - \frac{R_3}{R_i} V_i \ldots \]

Q: All weights are negative. How can we change sign?
Q: Can we add an inverting amp circuit?
A: Not always…. But perhaps here is OK.

Q: What’s the requirement on \( R_{th} \)?
A: \( R_{th} = 0? \)
Unity Gain Buffer

- Safely cascading circuit modules
Unity Gain Buffer

• Safely cascading circuit modules
Unity Gain Buffer

- Safely cascading circuit modules

\[ U^+ = V_{in} \]
\[ U^- = V_{out} \]

NFB ⇒ Golden Rule #2 ⇒ \[ U^- = U^+ \]

\[ V_{in} = V_{out} \]