Logistics: ① Discussions: Try different sections Mon AND Wed

Find a TA that works for you.

Less crowded: Shashank 8-7 am
Tseguda
Nishe
Niyuki 12-1 pm

② Homework 1 due on Friday 9/4.

③ Study group survey.

③ Office hours - pizza post is coming.

Linear equations

System of linear equations

\[ 2x + 3y = 8 \text{ (E1)} \]
\[ 3x - y = 1 \text{ (E2)} \]

How to solve systems with million of eq's + variable?
\[ 2x + 3y = 8 \quad (E1) \]
\[ 3x - y = 1 \quad (E2) \]

1. Normalize \( E1 \) so that the coefficient of \( x \) is 1.

\[ E1 \rightarrow (E1) / 2 \]
\[ x + \frac{3}{2}y = 4 \quad (E1\#) \]
\[ 3x - y = 1 \quad (E2) \]

\[ \begin{bmatrix}
2 & \frac{3}{2} & 8 \\
3 & -1 & 1
\end{bmatrix} \quad \text{Augmented matrix form.} \]

\[ \begin{bmatrix}
1 & \frac{3}{2} & 4 \\
3 & -1 & 1
\end{bmatrix} R1 \\
\begin{bmatrix}
1 & \frac{3}{2} & 4 \\
0 & -\frac{1}{2} & -11
\end{bmatrix} R2
\]

Take \((E2) - 3(E1\#)\)

\[(3x - y) - 3(x + \frac{3}{2}y) = 1 - 3 \times 4\]
\[-y - \frac{9}{2}y = -11\]
\[-\frac{11}{2}y = -11\]
\[-\frac{11}{2}y = -11 \quad (E2\#)\]

2. Solve for \( y \), by normalizing \((E2\#)\)

\[ y = 2 \]

3. Back substitute \( \rightarrow (E1\#) \)

\[ x + \frac{3}{2}y = 4 \]
\[ x + 3 = 4 \]
\[ x = 1 \]

\[ \begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 2
\end{bmatrix} \quad \text{Upper triangular matrix.} \]

4. Gaussian Elimination

Linear solve \( \rightarrow \) Ancient China
\[
\begin{bmatrix}
2 & 3 \\
3 & -1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
=
\begin{bmatrix}
8 \\
1
\end{bmatrix}
\]

Matrix \quad \text{vector} \quad \text{vector}.

\underline{\text{Vector: Ordered list of elements.}}
\begin{align*}
e.g. \quad \begin{bmatrix} 2 \\ 1 \end{bmatrix}, & \quad \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \\
& \quad \text{2D vector} \quad \text{3D vector}
\end{align*}

\vec{x} \quad \in \quad \mathbb{R}^2 \quad \text{2 dim of real numbers}
\quad \uparrow \\
\text{arrow m \rightarrow n} \\

\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}

\vec{y} \in \mathbb{R}^3 \\
\vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}

Matrix: Grid of numbers

\[
A = \begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6
\end{bmatrix}
\]

2x3 Matrix

\[A \in \mathbb{R}^{2 \times 3}\]

---

Gaussian Elimination:

\[
\begin{align*}
2x + 3y &= 8 \quad (E_1) \\
2x + 3y &= 6 \quad (E_2)
\end{align*}
\]

1. Normalize (R1)

\[
(R1) \leftarrow (R1)/2
\]

Want 0 here

2. Eliminate x from second row

\[
(Row 2) - 2 \times (Row 1)
\]

Equation

\[
0x + 0y = -2
\]
Conclude: No solution to this system.

In an upper triangular matrix, the diagonal entries are called pivots.

If you have a 0 on the diagonal, this means you have to be careful.

Example 3

\[ \begin{align*}
    x + 4y &= 6 \\
    2x + 8y &= 12
\end{align*} \]

\[ \begin{bmatrix}
    1 & 4 & 6 \\
    2 & 8 & 12
\end{bmatrix} \]

1) Normalize.
   Not needed
   Already done

2) Subtract 2(R1) from R2.

\[ \begin{bmatrix}
    1 & 4 & 6 \\
    0 & 0 & 0
\end{bmatrix} \]
Infinite number of solutions!

Geometric perspective:

\[ 2x + 3y = 8 \]

Intercepts: \((0, 8\frac{1}{3})\) \((4, 0)\)

\[ 3x - y = 1 \]

Intercepts: \((0, -1)\) \((\frac{1}{3}, 0)\)
Geometry for infinite solutions

Geometry of no solutions
Gaussian Elimination

\[
\begin{pmatrix}
    a_{11} & a_{12} & \cdots & a_{1m} & \bar{b}_1 \\
    a_{21} & a_{22} & \cdots & a_{2m} & \bar{b}_2 \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    a_{m1} & a_{m2} & \cdots & a_{mn} & \bar{b}_m
\end{pmatrix}
\]

\[
\Rightarrow a_{11}x_1 + a_{12}x_2 + \cdots + a_{1m}x_m = \bar{b}_1
\]

Every row represents a measurement.

Every column is a variable.

1. Start with row 1.
   - If \( a_{11} \) is 0, swap rows to have a non-zero coefficient for \( x_1 \) in row 1.

2. Normalize so that \( a_{11} \) has a coefficient of 1 in the first row.

3. Use first row to eliminate \( x_1 \) from all other equations/rows.

4. Move on to 2nd variable: Repeat with row 2.
5) Get to an upper triangular matrix
6) Back-substitute.