

#	Question	Answer(s)
1	is delta equals and := the same thing?	Yes they are the same (both mean 'is defined as'), but please try to use delta equals in this course so we are consistent in notations.
2	where can I find the recorded discussions? I couldn't find the link on the website.	They are posted on eecs16a.org in the 'Schedule' section, just below the link to the worksheets.
3	In the previous slide, does $M=N$ ?	Not necessarily.
4	what is $a_1$ ?	The first column of the matrix $A$ , written as a vector.
5	i didnt understand the part about the $b$ vector and being able to solve equations without taking measurements. How does this work?	The goal is to see if there is a linear combination of the columns of $A$ that gives us $b$ . Thinking of the columns of $A$ is different from the row perspective from last lecture, where they represented measurements.
6	is this an example of a linear combination?	Yes $b$ is a linear combination of $a_1$ and $a_2$
7	is gaussian elimination considered row or column view?	Technically neither. GE is a tool to solve the $Ax=b$ set up. You can describe it in terms of either view.
8	why didn't we stop after you got upper triangle?	We want to solve for $x_1$ and $x_2$ . You can stop at the upper triangular form and then solve for $x_1$ and $x_2$ by substitution.
9	how does the process of gaus. elimination represent doing a linear combination?	A linear comb is $Ax=b$ . GE is a way to solve $Ax=b$ by setting up the augmented matrix in the form $[A b]$ .
10	Why do values of 2 and 1 for $x_1$ and $x_2$ satisfy the equation?	Try plugging $x_1=2$ and $x_2=1$ back into the original $Ax=b$ setup.
11	So matrix $A = \text{vector } a_1 * \text{vector } a_2$ ?	No it's not a multiplication. $a_1$ and $a_2$ are just columns in the matrix $A$
12	do we always graph $a_1$ first?	Nope, either way is okay
13	How are we *solving* w.r.t. linear comb.ns? I get that the answer is a set of linear combinations, but didn't we solve it graphically?	We did solve it graphically, but we also want to show how to solve it algebraically, as defined by the linear combination.
14	when is the linear combo method better to use than the row method?	There are ways we can "embed" information in a linear combination. We will see this more later in this lecture and next.
15	are $x_1$ and $x_2$ (in Gaussian elim. matrix) in place of coefficients of $a_1$ and $a_2$ ? if so, does that mean the equation would be $x_1a_1 + x_2a_2 = b$ ? I'm a little confused	Yes the equation is $x_1a_1 + x_2a_2 = b$ . Remember $x_1$ $x_2$ are scalars and $a_1$ , $a_2$ , $b$ are vectors. $a_1$ and $a_2$ are still coefficients and $x_1$ $x_2$ are variables in Gaussian elimination.
16	What was the graphical method called (or does it have no name)?	Emmm I think you can just call it the graphical method...
17	We can also leave it in REF instead of RREF and solve that way right?	Yes
18	Does infinity count as a scalar?	technically yes. a scalar is a single number
19	does this only work if the $A$ vectors are perpendicular?	No, but in generally, we like when they are not parallel. We'll see this more shortly
20	is it possible to use the complex plane to move vectors toward the $b$ vector?	Technically yes. The complex plane is somewhat of an additional axis. it depends on the application whether or not we want to go there.
21	Can we use non-integer values for the scaling?	Yes!
22	what if they are not perpendicular?	live answered
23	what are the points on the $a_1$ v $a_2$ axes?	
24	so there are 4 different possibilities for $b$ ?	If you're asking about how many possibilities we have for $Ax=b$ , then there are infinitely many options for $b$ . It's all the different combinations of the columns of $A$ , depending on the scalars / coefficients in $x$ .
25	Why those lines acting like axes leads to concluding we can reach any points in 2D? Can you go over that again please?	we will try to visualize this again later, but algebraically, we are saying that we can write any point $(x_1, x_2)$ as a linear combination of the the columns of the matrix
26	What if the $a_1$ and $a_2$ are the same vector?	in that case we cannot reach all vectors in the 2D plane by linear combinations of $a_1$ and $a_2$
27	So basically, we can reach any other $b$ vector with linear combos of $a_1$ and $a_2$ as long as $x_1$ and/or $x_2$ are different from what we solved right?	Yes
28	what are the axes?	We're supposing that the columns of $A$ are a new set of axes
29	This class is really hard	We're sorry to hear you are struggling :( We do not mean this class to be difficult, though it may be some tricky new material. Please reach out and stop by office hours. We really want to help you learn!
30	So it always works unless the two vectors are parallel?	Yes that's correct. We will see that later in the 'span' section.
31	shouldn't the axis be $x_1$ , $x_2$ ?	$x_1$ and $x_2$ are the regular Cartesian coordinate system axes. Here we want to represent another vector by the $a_1$ and $a_2$ vectors so they become our axes in this particular case.
32	isnt it possible to reach $b$ with a combination of $a_1$ - $a_2$ instead of $a_1+a_2$ ?	$a_1$ and $-a_2$ are still on the same line, so we still cannot reach a vector out of that line

33	so are we going to say that the span of that matrix is limited only to any scalar of the column vectors?	The span of the *columns of the matrix* is limited to only any scalar/linear *combination* of the columns. but yes
34	does that line formed by $a_1$ and $a_2$ act as a one dimensional axis?	Yes!
35	what if $b$ is parallel to $a_1$ but not to $a_2$	Still do able. Then $x_1$ will be nonzero, and $x_2$ will be 0.
36	So we can scale a vector by a negative value?	Yup, that's okay!
37	So graphically, linear dependence is when the two lines are parallel?	Yes for 2 vectors in 2D that's correct.
38	but if the vector is at a certain angle lets say 30 it can only rotate 30 how can it reach every point?	In this context, 30 degree vector is still just a vector. Adding 2 of them does not make it a 60 degree vector, but a 30 degree vector that is 2x as long. Doing a rotation is a linear transform that we will get into later today or next week.
39	I dont understand what that means	
40	So we,Äre allowed to multiply vectors $A_1$ and $A_2$ by different scalars? To reach all points I mean	Yes
41	what is ie	It's latin for "in other words"
42	How do we know that it is $R^2$ ?	Vectors in $R^2$ are defined only by 2 values. Please see Monday's discussion notes.
43	what does it mean if the matrix spans the whole field?	That means that we can reach any point in the space with some linear combination of the vectors in our set, in this case, the columns of our matrix
44	do we need to know set notation for this course?	Yes there will be set notations in the materials like notes/homework.
45	will we have to understand this fancy notation for exams?	There will be set notations in notes/homework so you will have some practice to become more familiar with that. But typically we will also explain them in other words.
46	Is there any notation we should write preceding the condition like "where" or is it sufficient to just right it after the expression in setbuilder?	
47	can you go back over the set notation again?	live answered
48	what is the meaning of columns. do they only represent the coefficient of the same variable?	They can be, but that's closer to the row context. Think about the columns as directions we can travel. Then the linear combination says how to combine those directions over many "steps" to get to a desired point.
49	could you write the second example in set notation?	live answered
50	So if the vectors are not parallel, is the span always $R^2$ ?	Yes in the 2D case.
51	how do we write the set for the second example (the line)	live answered
52	What does the line $x_1 = x_2$ mean? What line is it?	live answered
53	What would the set notation be for the $x_1=x_2$ example?	live answered
54	so span is just another way to represent the matrix?	Span is one way to describe vectors. In this case, the vectors we care about are the matrix columns
55	so if we had this question, what form would we answer it in? would it just be all real numbers?	
56	does the concept of span also work for row vectors?	Yes! you can also determine the row span
57	Prev topic question: Is it still Gaussian Elim if you transform an augmented matrix into REF form and solve for it without backsubstituting?	If a question specifically asks you to do Gaussian elim, please try to reduce to the RREF form if there is a unique solution. If a question asks you to solve a system of equations without requiring Gaussian elim, you can stop at the REF form.
58	why is it alpha 1 and beta with no subscript	It is actually alpha (comma) beta
59	What would the set notation be for the second example where the answer is the line $x_1=x_2$ ?	live answered
60	How are we determining the span? Just by graphing?	We can define it algebraically as the linear combination of the vectors.
61	if $A$ has a lot of columns, are we expected to list every single one in set notation?	Not necessarily. Sometimes if you have a lot of vectors, you will find that some of them are linear combinations of others. Then you can skip those.

62	how would we write the span(A) for the first example we did? where we said we could reach any point?	If it can reach any point in 2D you can say $\text{span}(A) = \mathbb{R}^2$
63	are we allowed to say things like $\text{span}(A) = \{\alpha [1,1] + \beta [1,-1] \}$ such that $\alpha$ and $\beta$ are real numbers? ?	Yes!
64	is $\alpha$ and $\beta$ equal to $x_1$ and $x_2$ ?	$\alpha$ and $\beta$ can be any real numbers in this case.
65	how does the concept of span work for row vectors?	We can define the span of row vectors as the set of all linear combinations of the row vectors, similar to the column vector case.
66	Will the dimension of $\mathbb{R}$ be specified in set notation? Such as $\mathbb{R}^2$ , $\mathbb{R}^3$ , etc.	Yes. Here $\alpha$ and $\beta$ are both scalars (1D real numbers)
67	$\mathbb{R}^3$ means that the span is all the 3d vectors right?	$\mathbb{R}^3$ means the space of all vectors defined by 3 scalar values. The right set of vectors in $\mathbb{R}^3$ can span the entire space, not necessarily "all of them".
68	does it matter what order we mark the axis in?	For 3D axes there is a right-hand convention. You can read more about it in Wikipedia "Right-hand rule".
69	What would be the span if there were only two independent vectors in a 3D space?	The span would be a 2D space (a plane), but not $\mathbb{R}^2$ .
70	How would we figure out span without graphing?	span can also be thought of as all linear combinations of the linearly independent vectors.
71	so if there are 4 vectors it would be $\mathbb{R}^4$ and so on?	No, the number of elements within a vector defines if we are in $\mathbb{R}^3$ , $\mathbb{R}^4$ , etc. The number of vectors we have define what the dimension of the span is.
72	so span is like the range of the linear combination?	Yes
73	Is the zero vector a point?	Yes. Or you can think about it like a vector from $(0,0)$ to $(0,0)$ , with a length = 0.
74	So if a number (n) of column vectors (not multiples of each other) equals the number of unknowns (n unknowns), then it spans the whole field $\mathbb{R}^n$ ?	Only if all the vectors are linearly independent
75	In this definition, what does it mean that $Ax=b$ ? Does that mean that there are linear combinations?	In this context, yes. We're saying that $b$ is a linear combination of the columns of $A$ with the coefficients from the $x$ vector.
76	shouldn't it be in span on $x$ vector and not $A$ because $A$ is a scalar not a vector	No, here, $A$ is matrix composed of column vectors. its in the span of the columns of $A$ .
77	Do we use $\text{span}(A)$ or $\text{span}(\text{col}A)$ ?	Technically $\text{span}(\text{col}(A))$ is more correct ("span of the column space of $A$ "), but we're going to be a bit loose since we're generally talking about the column space.
78	whats the span of $\{a_1, a_2, 0\}$ ?	Will be the same as the span of $a_1, a_2$ .
79	if $\text{span}(A)$ doesn't include $b$ , does that mean there is no linear combo of column vectors of matrix $A$ that equal $b$ ?	Yes!
80	the prof said that $\{a_1\text{vector}, a_2\text{vector}, 0\text{vector}\}$ are always linearly dependent. Would you mind explaining this a bit more? Thanks!	The trick here is that, because of the definition of the linear (in)dependence, if the zero vector is in our set, it will always be linearly dependent. Because we can always write the zero vector as a nonzero sum of the zero vector.
81	So will we have proofs like this on hw or exams?	yes
82	sorry i missed it, what does it mean if $b$ isn't in the span of $\text{col}A$	It means that $b$ can't be written as a linear combination of the columns of $A$
83	How do we define Linear independence again?	for a set of vectors $v_1, v_2, \dots, v_n$ , if there exist scalars $a_1, a_2, \dots, a_n$ (at least one of them is not zero), such that $a_1v_1 + a_2v_2 + \dots + a_nv_n = 0$ , then the vectors $v_1, v_2, \dots, v_n$ are linearly dependent.
84	Are we supposed to write $\text{span}(A)$ or $\text{span}(\text{cols } A)$ ?	Technically $\text{span}(\text{col}(A))$ is more correct ("span of the column space of $A$ "), but we're going to be a bit loose since we're generally talking about the column space.
85	did she say $\{a_1, a_2, 0\}$ is always linearly dependent because 0 can always be reached with any coefficient?	We can always represent $0 = 0 \cdot a_1 + 0 \cdot a_2$ , or $0 \cdot a_1 + 0 \cdot a_2 + \text{any non-zero coefficient} \cdot 0 = 0$ , so that set of vectors is linearly dependent.
86	what is set $S$	For the problem, we defined the set $S$ as being the span / linear combination of those two vectors.
87	why exactly does that prove it again	We were trying to prove that we could write any $b$ in terms of linear combination of those vectors.
88	How do we know that we've completed the proof?	The proof is complete when we started with our assumptions / knowns and arrive at the desired conclusion through the logic steps.
89	Where did the $b_1 + b_2 / 2$ come from?	It came from doing the Gaussian elimination on $[A] b$
90	can we state that because $b_1, b_2$ were arbitrary, then $\alpha$ and $\beta$ are also arbitrary and hence are in $\mathbb{R}$ ?	That's not sufficient. We need to show that we can find the $\alpha$ and $\beta$ to give us $b$ .
91	So our proof for spanning is that we get the identity matrix after G.E.?	Not exactly. The proof comes from being able to find an $a_1$ and $a_2$ to get to an arbitrary $b$ .

92	What does the solution mean?	
93	in the future will it be enough to just show LI or LD for proofs?	Yes. anything that we've already proved in lecture, discussion, homework; you can assume is already known (though you have state / show it), so you don't to re-prove it
94	Is it good enough to end the proof with those equations, or should we explain why the equations make theorem true?	It's always a good idea to briefly explain what the equations mean in the proof
95	Is finding a formula for the scalars alpha and beta enough to state that they can span R2? I don't understand how formulas explicitly show a span.	It's a sufficient proof because we found alpha and beta to reach an arbitrary b vector in R2. Since we didn't make any assumptions about b, then we have achieved the span.
96	how did those 2 equations we got, prove the question?	It's a sufficient proof because we found alpha and beta to reach an arbitrary b vector in R2. Since we didn't make any assumptions about b, then we have achieved the span.
97	can we go over how the proof was concluded again?	
98	how is finding $\alpha = (b_1+b_2)/2$ and $\beta = (b_1-b_2)/2$ proves that all b vects belong to set S?	We showed that for any given vector b, we can find alpha and beta such that $b = \alpha * (1,1) + \beta * (1,-1)$ , so b is in the span of (1,1) and (1,-1)
99	Could you do an example for span of columns?	
100	Sorry, I meant span of rows	We won't focus much on the span of rows in this class, but if you're curious, drop by office hours.
101	What do the final equations mean in the proof? What do they intuitively indicate?	We showed that for any given vector $b = (b_1, b_2)$ , we can find alpha and beta such that $b = \alpha * (1,1) + \beta * (1,-1)$ , so b is in the span of (1,1) and (1,-1)
102	Are all vectors that are useless redundant?	To be clear, "useless" refers to not giving us new information (as meant by redundant), because it is a linear combination of others.
103	How do you see quickly that a set of vectors are linearly dependent?	Some quick checks: if there are more vectors than the dimension (say there are 4 vectors and each of them is in R3), they are linearly dependent. If there is 0 in the set of vectors, they are linearly dependent.
104	if we want to prove that 2 vectors are linearly dependent we need to prove that $c_1 * v_1 + c_2 * v_2 = 0$ . The rule expects us to have at least 1 constant to be different than 0. But why not ALL constant should be different than 0?	consider the R2 set: [1, 0], [0, 1], [2, 0]. The set is linearly independent, but you only need 1 nonzero constant
105	what's $\sum_{j \neq i}$ is for?	The summation means for all m except where $m=j$ . The j comes from the vector $a_j$
106	Do we need to know this notation? Or can we just ignore it and understand how it works?	You should develop a basic understanding of summation notation and the basic set notation we've used in class.
107	doesn't that mean any 1 vector can be linearly dependent, if the scalar we use is 0	We have to have a non-zero scalar as the coefficient. So if we have one vector and it's non-zero, then it's linearly independent. If we have one vector and it's 0, then it's linearly dependent.
108	why can't it equal to j?	We can't include $a_j$ because it's trivial to write a vector in terms of itself. We only want to use the other vectors.
109	what does m not equal to j mean under the summation	We can't include $a_j$ because it's trivial to write a vector in terms of itself. We only want to use the other vectors.
110	Could you repeat what the definition of lin dep was in relation to span?	Linearly dependence means that at least one vector exists in the span of the others.
111	Can we say "A set of vectors is linearly dependent if any vector in that set is a linear combination of the other, right? Thanks.	It's sufficient to say: a set of vectors is linearly dependent if [one] vector in that set is a linear combination of the other. If [any] vector in that set is a linear combination of the other, yes they are linearly dependent, and it's more than sufficient.
112	In def 2, don't we have to specify that the constants are not all zero? Otherwise, it would always satisfy the definition.	yes
113	Don't you have to specify at least one $a_j, 1 \leq j \leq M$ is nonzero?	yes
114	don't we need to specify that at least one coefficient should not be zero?	yes
115	Isn't there a clause that, "like at least 1 is, not 0"	yes
116	If we get rid of the vector that is within the span of the others, is the set of remaining vectors no longer linearly dependent?	Not necessarily. There may be more than 1 vector that is causing linearly dependence.
117	so the only way to prove linear independence, is to prove that something is not linear dependent?	We will see more theorems of linear independence in the future lectures.
118	Is a set containing only 1 vector always linearly dependent? Since we can get that same vector by multiplying it by 1	No, because in our definition of linearly dependence, we strictly skip over that vector in the summation. ( $m \neq j$ )
119	what if the zero vector is the only vector in the set? then would that not be linearly independent?	No because of the definition of linear dependence

120	So a set containing only 1 vector is always linearly independent?	As long as that one vector is not the 0 vector, then yes.
121	so was that lin dep or lin indep?	last example was LD
122	The answer to that 3 vector question was not independent right?	Correct, because the vectors are in $R^2$
123	why can we tell its linear dependent without writing out the linear combinations?	We didn't say it rigorously, but it has to do with our vectors being in $R^2$ . If we have already have 2 linearly independent vectors, then the 3rd one will always be linearly dependent.
124	so the last example was linearly dependent correct?	yes
125	the last example	
126	wait what was the quick way for the last example again?	
127	What if we set all the alpha values to 0 in definition 2? She said that wasn't a restriction but then wouldn't all sets be linearly independent by definition	live answered
128	so, if $n = 2$ , the three vectors would only be linearly independent if the first two were parallel?	If the first two are parallel, they are already linearly dependent
129	dont you have to say that at least one alpha is nonzero in the LD defin?	yes
130	For the 3 vector question that was not linearly independent, does this mean the third vector was redundant because the first two vectors already spanned $R^2$ ?	yes
131	at least one alpha must be non-zero for them to be lin dep right?	live answered