

Repeated eigenvalues:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$(A - \lambda I) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Ch. poly: $(2 - \lambda)^2 \Rightarrow \lambda = 2, \lambda = 2.$

are my eigenvalues.

Null $(A - \lambda I)$

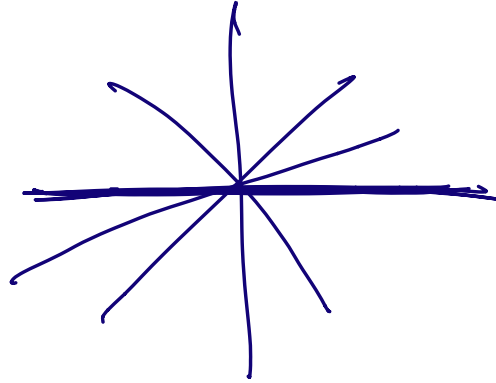
$$\text{Null}(A - 2I) = \text{Nul} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

↑ ↑

\mathbb{R}^2

$$A = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$



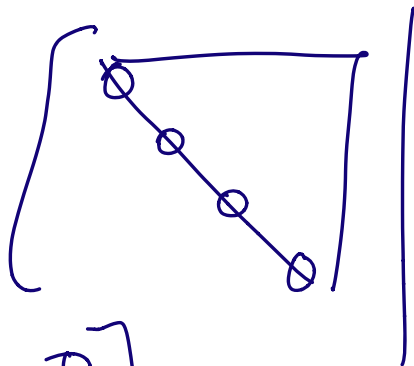
$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\text{Span}(A) = \text{Col}(A) = \text{Range}(A)$$

$$= \left\{ \vec{v} \mid \vec{v} = \alpha \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \alpha \in \mathbb{R} \right\}$$

$$A = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Pivot:



$$\begin{bmatrix} \textcircled{1} & 0 & 0 \\ 0 & \textcircled{1} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 4 & 0 \\ 2 & 5 & 0 \\ 3 & 6 & 3 \end{pmatrix}$$

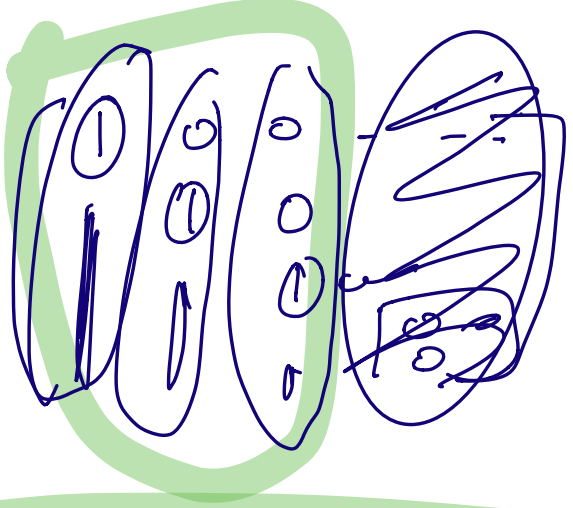
3x5 matrix

$$A = \begin{bmatrix} 1 & 4 & 7 & 1 & 4 \\ 2 & 5 & 9 & 2 & 2 \\ 3 & 6 & 7 & 3 & 3 \end{bmatrix}$$

~~span~~
basis

→ ① GE on A → []

~~free~~
basic → dimension of col. space.



A^T

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix}$$

$$\begin{aligned} \det(\) &= (1-\lambda)(4-\lambda) - 4 \\ &= 4 + \lambda^2 - 5\lambda - 4 \\ &= \lambda(\lambda - 5) \end{aligned}$$

e-values $\lambda=0$, $\lambda=5$



\vec{u}_1

$2\vec{u}_1$

$$A \vec{u}_1 = \lambda \vec{u}_1$$

$$A \cdot 2\vec{u}_1 = 2 \cdot A \cdot \vec{u}_1 = 2 \cdot \lambda \vec{u}_1 = \lambda (2\vec{u}_1)$$