Welcome to EECS 16A!
Designing Information Devices and Systems I

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Module 2
Lecture 7
Capacitive Touchscreens
(Note 17)
Last lecture: Capacitors

- Charge storage device (like a ‘bucket’ for charge)
- holds electric charge when we apply a voltage across it, and gives up the stored charge to the circuit when voltage removed

Symbol: \[ \cap \]  
Capacitance: \( C \)  
Units: Farads [F]

IV equation:
Capacitance

\[ C = \varepsilon \frac{A}{d} \]

\[ [F] = \left[ \frac{F}{m} \right] \left[ \frac{m^2}{m} \right] \]

Depends on:
- Materials: \( \varepsilon \) permittivity
  \[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \]
  \[ \varepsilon = \varepsilon_0 \varepsilon_r \]
- Geometry of Conductors

Conductive plates

Dielectric

Symbol:

\[ \frac{\phantom{A}}{} \]

Capacitance: \( C \)
Units: Farads [F]

IV equation:

\[ I = C \cdot \frac{dV}{dt} \]
Circuit Model: IV relationship

Capacitor Symbol

\[ Q_{\text{elem}} = C \cdot V_{\text{elem}} \]
\[ [C] \quad [F] \quad [V] \]
(Farad)

We know: \[ I_{\text{elem}} = \frac{d Q_{\text{elem}}}{dt} \]
\[ I_{\text{elem}} = C \cdot \frac{d V_{\text{elem}}}{dt} \]
\[ C = \text{constant over time} \]

→ Can use the same 7-step analysis.
Simple Circuit 1

KCL: \( I_s = I_c \)

Element Def.: \( I_c = C \cdot \frac{dV_c}{dt} \)

Voltage Def.: \( U_i - 0 = V_c \)

\[ I_s = C \frac{dU_i}{dt} \times dt \]

\[ I_s \cdot dt = C dU_i \]

\[ \int_0^t I_s \, dt = \int_0^t C \cdot dU_i \]

\[ I_s \cdot t = C \cdot (U_i(t) - U_i(0)) \]

\[ U_i(t) = \frac{I_s}{C} \cdot t + U_i(0) \]
Simple Circuit 2

\[
\begin{align*}
U_i - 0 &= V_s \quad \{ \text{Voltage Def.} \} \\
U_i - 0 &= V_c \\
V_s &= V_c \\
I_c &= C \frac{dV_c}{dt} \quad \{ \text{capacitor Def.} \} \\
I_c &= C \frac{dV_c}{dt} = C \cdot \frac{dV_s}{dt} = 0
\end{align*}
\]

Current in a capacitor is zero when a constant voltage source is across it.

**Hint:** We like zeros... they make our lives easier!
Simple Circuit 3

looking for $U_1$ value when

$V_c = \text{const. (steady-state)}$

$I_c = C \frac{dV_c}{dt} = 0$

KCL: $I_c^0 + I_R = 0$

$I_R = 0$

Ohm's law: $V_R = I_R R = 0$

Voltage Def: $U_1 - 0 = V_R^0$

$U_1 = 0$

Steady State: means the Voltages Settled.

If current is zero $\Rightarrow$ OPEN-CIRCUIT
Equivalent Circuits with Capacitors

* Capacitor-only circuits

Step 1: Find \( V_{th} \) and \( I_{no} \) no source

Step 2: \( C_{eq} = \frac{I_{elem}}{\frac{dV_{elem}}{dt}} \)

\[ \begin{array}{c}
\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = C_{eq} \\
\end{array} \]

Only if \( \left( \text{match } \frac{dV_{elem}}{dt} \right) \)
Two Methods:

a) Apply $I_{test}$ and measure $\frac{dV_{test}}{dt}$

b) Apply $\frac{dV_{test}}{dt}$ and measure $I_{test}$

\[ C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} \]

(a)
Example 1

\[ V_{c1} = U_1, \; V_{c2} = U_1 \quad \text{and} \quad U_1 = V_{\text{test}} \]

\[ \frac{dU_1}{dt} = \frac{dV_{\text{test}}}{dt} \]

Element def: \[ I_{c1} = C_1 \frac{dV_1}{dt} = C_1 \frac{dU_1}{dt} = C_1 \frac{dV_{\text{test}}}{dt} \]

Element def: \[ I_{c2} = C_2 \frac{dV_2}{dt} = C_2 \frac{dU_1}{dt} = C_1 \frac{dV_{\text{test}}}{dt} \]

KCL: \[ I_{\text{test}} = I_{c1} + I_{c2} = C_1 \frac{dV_{\text{test}}}{dt} + C_2 \frac{dV_{\text{test}}}{dt} \]
\[ I_{test} = (C_1 + C_2) \frac{dV_{test}}{dt} \]

\[ C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = C_1 + C_2 \]

\[ R_1 \quad \parallel \quad R_2 \]

\[ R_{eq} = R_1 + R_2 \quad \text{(Series)} \]
Example 2: "Capacitors in series"

KCL: \[ I_{C_1} = I_{C_2} = I_{test} \]

Elements:
\[ I_{C_2} = C_2 \frac{dV_{C_2}}{dt} \]
\[ I_{C_1} = C_1 \frac{dV_{C_1}}{dt} \]

Voltage Defn:
\[ V_{C_2} = U_2 - 0 \]
\[ V_{C_1} = U_1 - U_2 \]
\[ V_{test} = U_1 - 0 \]

For \( V_{C_2} \):
\[ I_{C_2} = C_2 \frac{dV_{C_2}}{dt} \]
\[ I_{test} = C_2 \frac{dU_2}{dt} = \frac{dU_2}{dt} = \frac{I_{test}}{C_2} \]

For \( V_{C_1} \):
\[ I_{C_1} = C_1 \frac{dV_{C_1}}{dt} \]
\[ \frac{dV_{U_1}}{dt} = \frac{I_{C}}{C_1} = \frac{dU_1 - dU_2}{dt} = \frac{I_{test}}{C_1} \]
\[ \frac{dU_1}{dt} = \frac{dU_2}{dt} + \frac{I_{test}}{C_1} = \frac{I_{test}}{C_2} + \frac{I_{test}}{C_1} \]

\[ \frac{dV_{U_1}}{dt} = \frac{dV_{test}}{dt} = I_{test} \left( \frac{1}{C_2} + \frac{1}{C_1} \right) \]

\[ C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} = C_1 || C_2 \]

\[ C_{eq} = C_1 || C_2 \quad (|| \quad parallel \quad mathematical \quad operator) \]
Example 3

\[ C_{eq} = C_1 \parallel (C_2 + C_3) \]

\[ \Rightarrow C_{eq_1} = C_2 + C_3 \]

\[ C_{eq} = C_1 \parallel C_{eq_1} \]
Capacitive Touchscreen – Model without touch

\[ C_0 = \varepsilon \cdot \frac{A}{d} \]
Capacitive Touchscreen – Model with touch

When there is a touch, it makes a capacitor!

Problem: How can Voltage/Current when the finger is one of the terminals?

Solution: Models / Good architecture
when no touch:

when touch:

Extra Capacitance due to touch!

Equivalent capacitance for $C_1$ in series with $C_2$

We only have access to nodes $e$ and $g$, not $f$

Redraw to focus on terminals (nodes) $e$ and $g$

$\Rightarrow$ Equivalent capacitance for $C_0$ in parallel to $\frac{C_1C_2}{C_1+C_2}$