Welcome to EECS 16A!
Designing Information Devices and Systems I

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Module 2
Lecture 6
Capacitors
(Note 16)
Now that we understand 2D resistive touchscreen, let's change it!

Circuit model for each resistive sheet is a grid of resistors.

Real-world touchscreens are usually capacitive, not resistive:
- don’t need to be flexible
- multi-touch is easier
- more sensitive
- increased contrast on screen
Now, Capacitors!

- Charge storage device (like a ‘bucket’ for charge)
The Physics of a Capacitor

* Energy is needed to move charge.

→ No current across the capacitor plates

→ Voltage Source provides Energy needed for flow of charges (e−)

https://www.youtube.com/watch?v=X4EUsTwZ110
The Physics of a Capacitor

Once the switch is ON, e⁻ flow!

https://www.youtube.com/watch?v=X4EUwTwZ110
The Physics of a Capacitor

Lack of electrons means holes! $h^+$

$t_2$

$t_3$

Potential difference between the two plates! $V$

Electric Field

https://www.youtube.com/watch?v=X4EUwTwZ110
The Physics of a Capacitor

Every Capacitor can be charged up to a fixed Voltage.

The capacitor will charge a "load" until the charges on the plate are equalized. (No change in V)

https://www.youtube.com/watch?v=X4EUwTwZ110
Charge storage device (like a ‘bucket’ for charge)
Holds electric charge when we apply a voltage across it, and gives up the stored charge to the circuit when voltage removed
Circuit Model: IV relationship

Capacitor Symbol

\[ Q_{\text{elem}} = C \cdot \frac{d}{dt} V_{\text{elem}} \]

\[ [\text{C}] \quad [\text{F}] \quad [\text{V}] \]

(Farad)

We know: \[ I_{\text{elem}} = \frac{dQ_{\text{elem}}}{dt} \]

\[ I_{\text{elem}} = \frac{d}{dt} C \cdot V_{\text{elem}} \]

\[ C = \text{constant over time} \]

\[ I_{\text{elem}} = C \cdot \frac{dV_{\text{elem}}}{dt} \]

\[ \Rightarrow \text{Can use the same 7-step analysis.} \]
Capacitance

\[ C = \varepsilon \frac{A}{d} \]

\[ [F] = \left[ \frac{E}{m} \right] \left[ \frac{m^2}{m} \right] \]

Depends on:
- Materials: \( \varepsilon \) permittivity
  \[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \]
  \[ \varepsilon = \varepsilon_0 \varepsilon_r \]
- Geometry of Conductors

Conductive plates

Symbol:

Capacitance: \( C \)
Units: Farads [F]

IV equation:
\[ I = C \cdot \frac{dV}{dt} \]
**Simple Circuit 1**

![Circuit Diagram]

KCL: \( I_S = I_c \)

Element Def.: \( I_c = C \cdot \frac{dV_c}{dt} \)

Voltage Def.: \( V_i - 0 = V_c \)

\[ I_s = C \frac{dU_i}{dt} \times dt \]

\[ I_s \cdot dt = C \frac{dU_i}{dt} \]

\[ \int_0^t I_s dt = \int C \cdot dU_i \]

\[ U_i(t) - U_i(0) \]

\[ I_s \cdot t = C \cdot (U_i(t) - U_i(0)) \]

\[ U_i(t) = \frac{I_s}{C} \cdot t + U_i(0) \]
Simple Circuit 2

\[ V_s - 0 = V_c \] \quad \text{(Voltage Def.)}

\[ V_s = V_c \]

\[ I_c = C \frac{dV_c}{dt} \quad \text{(capacitor Def.)} \]

\[ I_c = C \frac{dV_c}{dt} = C \cdot \frac{dV_s}{dt} = 0 \]

Current in a capacitor is zero when a constant voltage source is across it.

Hint: We like zeros... they make our lives easier!
Simple Circuit 3

Looking for $U_i$ value when $V_c = \text{const. (steady-state)}$

$$I_C = C \frac{dV_C}{dt} = 0$$

**KCL:** $I^o_C + I_R = 0$

$I_R = 0$

**Ohm's law:** $V_R = I^o_R R = 0$

**Voltage Def:** $U_i - 0 = V_R^o$

$U_i = 0$

Steady State: means the Voltages settled.

If current is zero $\Rightarrow$ Open Circuit
Equivalent Circuits with Capacitors

- Capacitor-only circuits

Step 1: Find $V_{in}$ and $I_{no}$ no source

Step 2: $C_{eq} = \frac{I_{elem}}{\frac{dV_{elem}}{dt}}$

![Diagram of equivalent circuits with capacitors]
Two Methods:

a) Apply $I_{test}$ and measure $\frac{dV_{test}}{dt}$

b) Apply $\frac{dV_{test}}{dt}$ and measure $I_{test}$

$$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}}$$

(a)
Example 1

\[ V_{C_1} = U_1, \quad V_{C_2} = U_1 \quad \text{and} \quad U_1 = V_{\text{test}} \]

\[ \frac{dU_1}{dt} = \frac{dV_{\text{test}}}{dt} \]

Element definition:

\[ I_{C_1} = C_1 \frac{dV_1}{dt} = C_1 \frac{dU_1}{dt} = C_1 \frac{dV_{\text{test}}}{dt} \]

\[ I_{C_2} = C_2 \frac{dV_2}{dt} = C_2 \frac{dU_1}{dt} = C_1 \frac{dV_{\text{test}}}{dt} \]

KCL:

\[ I_{\text{test}} = I_{C_1} + I_{C_2} = C_1 \frac{dV_{\text{test}}}{dt} + C_2 \frac{dV_{\text{test}}}{dt} \]
\[ I_{\text{test}} = (C_1 + C_2) \frac{dV_{\text{test}}}{dt} \]

\[ C_{eq} = \frac{I_{\text{test}}}{dV_{\text{test}}/dt} = C_1 + C_2 \]

\[ R_{eq} = R_1 + R_2 \]

Series
Example 2: "Capacitors in series"

\[
\begin{align*}
&\text{KCL: } I_{c_1} = I_{c_2} = I_{\text{test}} \\
&\text{Elements:} \\
&I_{c_2} = C_2 \frac{dV_{c_2}}{dt} \\
&I_{c_1} = C_1 \frac{dV_{c_1}}{dt} \\
&\text{Voltage Def:} \\
&V_{c_2} = U_2 - 0 \\
&V_{c_1} = U_1 - U_2 \\
&V_{\text{test}} = U_1 - 0
\end{align*}
\]

For \( V_{c_2} \):
\[
I_{c_2} = C_2 \frac{dV_{c_2}}{dt}
\]
\[
I_{\text{test}} = C_2 \frac{dU_2}{dt} = \frac{dU_2}{dt} = \frac{I_{\text{test}}}{C_2}
\]

For \( V_{c_1} \):
\[
I_{c_1} = C_1 \frac{dV_{c_1}}{dt}
\]
\[
\frac{dV_i}{dt} = \frac{I_c}{C_1} = \frac{dU_i - dU_2}{dt} = \frac{I_{\text{test}}}{C_1}
\]
\[
\frac{dU_i}{dt} = \frac{dU_2}{dt} + \frac{I_{\text{test}}}{C_1} = \frac{I_{\text{test}}}{C_2} + \frac{I_{\text{test}}}{C_1}
\]
\[
\frac{dV_i}{dt} = \frac{dV_{\text{test}}}{dt} = I_{\text{test}} \left( \frac{1}{C_2} + \frac{1}{C_1} \right)
\]
\[
C_{\text{eq}} = \frac{I_{\text{test}}}{\frac{dV_{\text{test}}}{dt}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} = C \ll C_2
\]
\[
C_{\text{eq}} = C \ll C_2 \quad (\ll = \text{parallel mathematical operator})
\]
Example 3

\[ C_{eq} = C_1 \parallel (C_2 + C_3) \]

\[ \Rightarrow C_{eq_1} = C_2 + C_3 \]

\[ C_{eq_2} = C_1 \parallel C_{eq_1} \]
Capacitive Touchscreen – Model without touch

\[ C_0 = \varepsilon \cdot \frac{A}{d} \]
Capacitive Touchscreen – Model with touch

When there is a touch, it makes a capacitor!

- finger
- dielectric
- conductive plate

Problem: How can Voltage/Current when the finger is one of the terminals?

Solution: Models / Good architecture
When no touch:

\[ e \quad \frac{1}{C_0} \quad g \]

With touch:

\[ e \quad \frac{1}{C_0 + \frac{C_1C_2}{C_1+C_2}} \quad g \]

We only have access to nodes \( e \) and \( g \), not \( f \).

Redraw to focus on terminals (nodes) \( e \) and \( g \).

Equivalent capacitance for \( C_1 \) in series with \( C_2 \):

\[ \frac{C_1C_2}{C_1+C_2} \]

\[ \Rightarrow \text{Equivalent capacitance for } C_0 \text{ in parallel to } \frac{C_1C_2}{C_1+C_2} \]