Welcome to EECS 16A!
Designing Information Devices and Systems I

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Module 2
Lecture 7
Capacitors and Capacitive Touchscreens
(Note 17)
Greetings from Miki & Ana
Last lecture: Capacitors

- Charge storage device (like a ‘bucket’ for charge)
- holds electric charge when we apply a voltage across it, and gives up the stored charge to the circuit when voltage removed

Symbol: \[ \cap \]

Capacitance: \( C \)  
Units: Farads [F]

IV equation: \[ I = C \cdot \frac{dV}{dt} \]

Conductive plates

\[ C = \varepsilon \cdot \frac{A}{d} \]
Circuit Model: IV relationship

Capacitor Symbol

\[ Q_{\text{elem}} = C \cdot V_{\text{elem}} \]

\[ [C] \begin{bmatrix} C \end{bmatrix} = [F] \begin{bmatrix} V \end{bmatrix} \]

(Farad)

We know: \( I_{\text{elem}} = \frac{dQ_{\text{elem}}}{dt} \)

\[ I_{\text{elem}} = \frac{d}{dt} C \cdot V_{\text{elem}} \]

\( C = \text{constant over time} \)

\[ I_{\text{elem}} = C \cdot \frac{dV_{\text{elem}}}{dt} \]

→ Can use the same 7-step analysis.
Equivalent Circuits with Capacitors

* Capacitor - only circuit

Step 1: find $V_{th}/Ino$  no source

Step 2: $C_{eq} = \frac{I_{eq}}{\frac{dV_{el}}{dt}}$

\[
\begin{align*}
\text{Step 1:} & \quad \text{find } V_{th}/Ino \quad \text{no source} \\
\text{Step 2:} & \quad C_{eq} = \frac{I_{eq}}{\frac{dV_{el}}{dt}}
\end{align*}
\]
a) Apply \( I_{test} \) and measure \( \frac{dV_{test}}{dt} \) and

\[
C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}}
\]

b) Apply \( \frac{dV_{test}}{dt} \) and measure \( I_{test} \)

* These are methods for experiments
Elem Definition: \( I_{c_1} = C_1 \frac{dV_{c_1}}{dt} \)

\( I_{c_2} = C_2 \frac{dV_{c_2}}{dt} \)

KCL: \( I_{test} = I_{c_1} + I_{c_2} = C_1 \frac{dV_{test}}{dt} + C_2 \frac{dV_{test}}{dt} = (C_1 + C_2) \frac{dV_{test}}{dt} \)

\[ V_{c_1} = U_1, \quad V_{c_2} = U_1, \quad U_1 = V_{test} \]

\[ \frac{dU_1}{dt} = \frac{dV_{test}}{dt} \]
\[ I_{\text{test}} = (C_1 + C_2) \frac{dV_{\text{test}}}{dt} \]

\[ C_{\text{eq}} = \frac{I_{\text{test}}}{dV_{\text{test}}/dt} = C_1 + C_2 \]  

[parallel]
KCL: $I_{c_1} = I_{c_2} = I_{test}$

Elem Definition:

$I_{c_2} = C_2 \frac{dV_{c_2}}{dt}$

$I_{c_1} = C_1 \frac{dV_{c_1}}{dt}$

$V_{c_2} = U_2 - 0 = U_2$

$V_{c_1} = U_1 - U_2$

$V_{test} = U_1$
For $V_{C_2}$: \[ I_{test} = I_{C_2} = C_2 \frac{dU_2}{dt} \Rightarrow \frac{dU_2}{dt} = \frac{I_{test}}{C_2} \]

For $V_{C_1}$: \[ I_{C_1} = C_1 \frac{dU_1}{dt} - \frac{dU_2}{dt} \Rightarrow \frac{I_{test}}{C_1} = \frac{I_{test}}{C_1} = \frac{dU_1 - dU_2}{dt} \]

\[
\frac{dU_1}{dt} = \frac{I_{test}}{C_1} + \frac{dU_2}{dt} \quad \text{Substitute} \quad \frac{dU_1}{dt} = \frac{I_{test}}{C_1} + \frac{I_{test}}{C_2}
\]

\[
\frac{dV_{test}}{dt} = I_{test} \left( \frac{1}{C_1} + \frac{1}{C_2} \right)
\]

\[
C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{C_1}{C_2} \quad \text{Series}!
\]
Equivalent capacitors

Capacitors in **Series**

\[ C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \]

Capacitors in **Parallel**

\[ C_{eq} = C_1 + C_2 \]
$C_{eq} = C_1/11 (C_2 + C_3)$
Capacitive Touchscreen – Model without touch

\[ C_0 = \varepsilon \frac{A}{d} \]

\[ \int_{E_2} \frac{E_1}{C_0} \]
Capacitive Touchscreen – Model with touch

When there is touch, we form a capacitor:

Problem: How can we measure $V$ or $I$ if our electrode is a finger?
When no touch

\[ E_1 \quad \text{=} \quad C_0 \quad \text{=} \quad E_2 \]

Circuit model:

Redraw to focus on terminals we can measure.
No touch

With Touch

\[ C_0 \quad E_1 \]

\[ C_0 + \frac{C_1 C_2}{C_1 + C_2} \]

\[ E_2 \]

\[ \frac{C_1 C_2}{C_1 + C_2} = C_0 \text{ (change)} \]
How do we measure change in capacitance?

**Option 1:**

Assume $V_{out}(0) = 0$

$$I_s = C_q \frac{dV_{out}(t)}{dt} \Rightarrow V_{out}(t) = \int_0^t \frac{I_s}{C_q} dt$$

$$V_{out} = \frac{I_s \cdot t}{C_q} \Rightarrow C_q = \frac{I_s \cdot t}{V_{out}}$$

Can't build current source easily.
Measuring Capacitance Models – Attempt #1

If there is touch: \( V_{out} = V_s \)

If there is no touch: \( V_{out} = V_s \)

Bad Idea!
Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor

1. Close both switches
   
   Same as before

2. Phase 1: Close $s_1$, open $s_2$

\[ Q = V_s \cdot C_{eq} \]
Measuring Capacitance Models – Attempt #2 – add switches and a reference capacitor

Initial condition?

Charge sharing

Phase 2: Close S2, open S1
Measuring Capacitance Models – Attempt #3 – known initial condition

Phase 1: $S_1$, $S_3$ closed, $S_2$ open
- Charge $C_{eq}$
- Discharge $C_{ref}$

$Q_{ref} = C_{ref} \cdot V_{out} = 0$ \hspace{1cm} \footnotesize{(V_{out}=0)}

$Q_{eq} = C_{eq} \cdot V_s$
Measuring Capacitance Models – Attempt #3 – known initial condition

redistribute, until same voltage

Phase 2: $S_1$, $S_3$ open, $S_2$ closed
Voltage across $C_{eq}$: $V_{out}$
Voltage across $C_{ref}$ = $V_{out}$

$Q_{total,2} = C_{eq} \cdot V_{out} + C_{ref} \cdot V_{out}$
Effect of touch on total capacitance

Total charge is conserved!

\[ Q_{\text{total,1}} = Q_{\text{total,2}} \]

\[ C_{\text{eq}} \cdot V_s = C_{\text{eq}} \cdot V_{\text{out}} + C_{\text{ref}} \cdot V_{\text{out}} \]

\[ V_{\text{out}} = \frac{C_{\text{eq}}}{C_{\text{eq}} + C_{\text{ref}}} \cdot V_s \]

By touching, we change voltage
Effect of touch on total capacitance

\[
\begin{align*}
\text{no touch} & : E_1 \quad C_0 \quad V_{out} = \frac{C_0}{C_0 + C_{ref}} \cdot V_s \\
\text{with touch} & : E_1 \quad C_0 + \frac{C_1 C_2}{C_1 + C_2} \quad V_{out} = \frac{C_0 + C_0}{C_0 + C_0 + C_{ref}} \cdot V_s
\end{align*}
\]
How can we go from voltage measurement to binary answer: touch or no touch?

- Threshold Voltage ($V_{th}$):
  - Between $V_{out\_touch}$ & $V_{out\_no\_touch}$
  - Above $V_{th}$: 1 (touch)
  - Below $V_{th}$: 0 (no touch)

We need to compare $V_{out}$ to $V_{th}$

So far: △ △ △ △ △ △ 🎀 🎀
How can we go from voltage measurement to binary answer: touch or no touch?

- New tools are needed – new circuit elements

![Circuit Elements Diagram]
An example of an Op-amp circuit diagram

Schematic diagram of a model 741 op-amp.
An op-amp (operational amplifier) is a device that transforms a small voltage difference into a very large voltage difference.

An op-amp has two input terminals marked (+) and (−) with potentials $U_+$ and $U_-$, two power supply terminals called VDD and VSS, and one output terminal with potential $U_{out}$.
Comparator – optimized for binary output & speed

\[ V_{\text{out}} = V_{\text{DD}} \text{ if } V^* > V_{\text{DD}} \]

\[ V_{\text{out}} = V_{\text{SS}} \text{ if } V^* < V_{\text{SS}} \]

Assume \( A = \infty \)
Comparator – optimized for binary output

If \( V_{\text{eit}} > V_{\text{th}} \), \( V_{\text{out}} = V_{\text{DD}} \)

If \( V_{\text{c}(t)} \leq V_{\text{th}} \), \( V_{\text{out}} = V_{\text{SS}} \)
Back to our Capacitive Touchscreen

In the diagram, the voltage $V_{DD}$ is connected to the capacitors $C_{eq}$ and $C_{ref}$, and the output $V_{comp}$ is compared against a threshold voltage $V_{th}$. The circuit indicates that when a touch occurs, the voltage $V_{DD}$ is applied to the capacitors, and no touch occurs when the voltage remains at $V_{SS}$. The diagram also shows that the output is expected to be between $V_{touch}$ and $V_{notouch}$. The circuit uses an operational amplifier to compare the voltages and determine the touch state.
Enjoy Spring Break!