Welcome to 16A!

If you’re like many students, you might find the official catalog title of this course “Designing Information Devices and Systems” a bit confusing. Why are “information devices and systems” important?

If this were the 17th century, medical imaging would be something like this:

![Image: Figure 1: Rembrandt, “The Anatomy Lesson of Dr. Nicolaes Tulp,” 1632 (with modifications).]

The only way to see inside of the human body was to actually look inside of the human body with human eyes. Thankfully, today non-invasive imaging tools such as Magnetic Resonance Imaging (MRI), CT scanners, X-ray machines, and ultrasound devices exist that can provide much of this information. In other words, we have devices, such as imagers, that provide information, such as a visualization of the human body. Often, these devices don’t work alone — they are part of a larger system that uses a combination of both physical sensors and signal processing techniques. Today, these “information devices and systems” are everywhere. For instance, smartphones take user input from touchscreens, microphones, accelerometers, and GPS to let you search the web, share pictures, communicate with others, and tell you where you are.

The goal of this class is not just to teach you the basics of linear algebra or the basics of circuits, but to help show you how those concepts weave together to enable the complex devices we use today. Of course, doing this will require helping you discover both basic circuits and linear algebra concepts. Without circuits to

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interface with the real world, these devices wouldn’t be very useful, and without processing techniques, we wouldn’t be able to extract meaningful information from circuit measurements. What is less obvious to an outside observer is that the ways of thinking required to understand both sides are actually similar, and this manifests in the mathematical language that is used. You might have heard buzz or hype around the term “machine learning” before taking this class — it turns out that too is another branch of the same tree. Our goal in 16A is to make it clear to you that all of the concepts you will learn about have a direct real-world application (and, in many cases, a multitude of applications). In order to put these concepts together to build a working system, we need to understand design thinking.

0.1 What this course is like and how you can succeed

Before going more into the actual subject matter of this course, it is useful to step back and reflect on the nature of this course at a higher level. The EECS 16AB series is fundamentally about teaching you how to think like an engineer while making you stronger and more mathematically mature. The primary mathematical vehicle to do this is linear-algebra (and related concepts) with the goal of you making these tools your own through your own hard work. This is done by showing you how you could have come with all the definitions yourself, as well as all the key proofs/derivations, and actually making you do many of these yourself — including deriving things and working through ideas seemingly not explicitly discussed in lecture/discussion, in ways that exercise the ways of thinking that are demonstrated and taught to you in lecture and discussion.

While mathematical curiosity is going to be fostered at points, the style is largely what is called “use-inspired” (this comes from Pasteur), where we show how each of the ideas emerge organically from engineering contexts, while being useful beyond the specific contexts that first motivated them. Our promise to you is that you will always know why you are learning whatever it is that we are teaching you.

0.1.1 Why is this course this way?

[Note to students: feel free to skip this section and just come back to read it whenever you find yourself wondering why things are the way that they are.]

This course is likely to feel very different to you. To understand why it is the way that it is, let us step way back for a moment and consider the role of education itself. Simply put, the goal of the educational system is to help students grow and become productive members of society. Our society is the inheritor of many thousands of years of human civilization, and somehow, we need to take approximately 24 years of formal and informal education to get you to take your rightful place on the front lines of society and then eventually as leaders who can then hand our civilization to the generations that will come after you.

Obviously, it is not possible to have everyone know everything. At some point, the system starts specializing more and more so that it is actually possible for the student to realistically grow towards some kind of mastery of something. Society would not be served if education merely produced a huge crop of dilettantes. Undergraduate education is one of the points at which people start to specialize, and if you are in this course, you suspect that Engineering (broadly understood) might be something that you want to be involved with more deeply.

\[2\text{If you ever find yourself losing track of this, just ask. Usually the lectures, discussions, labs, notes, and problems will try to make it clear. But if it isn’t clear to you, just ask. The course staff will help you.}\]
If you are a first-year undergraduate student, you are basically starting year 14 of this educational process. Let us think of someone who has just graduated with their PhD\(^3\) as being at the end of year 24 of this process. It can be hard to think about what you might want to be like approximately ten years into the future. You will be a different person, and one of the things that we are most blind to as human beings is just how much change and growth is possible on the scale of decades.

So, how can you even begin to get an intuitive grasp of the scale of change in the future? One way is to think about the past. Go back ten years in your education and reflect upon yourself as someone in elementary school. Hopefully you can see that that person, although they had a lot of potential, clearly had a lot to learn and a lot more mental growth ahead of them. It is hopefully also clear that whatever thoughts or opinions that person had about their education or the nature of the world, those thoughts were constrained and limited by the realities of being an elementary school student. Of course, there is nothing wrong with that — it is simply the nature of this world. It should also be clear that the difference between you now and you in elementary school is not merely that you know more facts now — you can think differently now. Once you have begun to grasp the gap between you now and your elementary school self, it is useful to understand yourself as virtually being in elementary school when compared to the world that the faculty and PhD students exist in. There is a lot to learn.

EECS 16AB are designed to help you transition to the kind of thinking that you are going to need as a professional. Essentially, for the majority of your working lives, you will be constantly encountering situations that are, to varying degrees, unknown to you. You will have to learn in-situ and make progress. There will not be a teacher there to point out what you need to do and if you are the kind of leader that we hope you will grow into, there also will not be a website or book out there that will tell you what you need to do. You will find yourself exposed to ideas in one context, and will have to actually weave them into what you do in a different context. For many of you, this might be different from what has been the dominant experience in your education before coming to Berkeley.

Young children are sometimes compared to sponges when it comes to education — they soak things in rapidly. Consequently, uncritical thinking and learning a vast amount of patterns often defines early education. This is also what is most useful for taking standardized exams like the SAT and becomes very important in your early education as a result. However, to succeed as an engineer, you will have to take more ownership of your mental models. You may have to change what you think of as understanding itself. In the past, it might have been possible to solve problems by looking in the textbook or online for a very similar problem or example, and then figuring out how to apply the same approach to the problem you were facing. This approach needs to be expanded on in your undergraduate career and beyond. Search engine skills will not get you all the way. We need you to be able to approach new problems sincerely and systematically, without necessarily having any easy pattern to match to or formulas to apply.

In Sanskrit poetics, the swan/hamsa bird is said to be imbued with the ability to drink milk that has been mixed into mud. This is a classical image used to convey the idea of being able to discriminate between what is important and what is not. As a student, you will be learning to discriminate as well. However, you need to be aware that some of your previous instincts could lead you astray. For example, some you might be used to skimming a mathematical text or listening to a lecture and taking care to note down key definitions or key formulas and examples of their use. In the context of EECS 16AB (and most of the later courses that you will take), this is almost always not very helpful and represents leaving much of the figurative milk behind.

\(^3\)Of course, not everyone is going to do a PhD or pursue any kind of formal graduate degree. However, the first few years of working in Industry typically involve a lot of informal training/mentorship and picking up skills as well as ways of thinking — this plays a similar role as graduate school for society in terms of bringing people up to speed and making them ready to take their places on the front lines. This is why so many jobs require significant experience.
What is really important to understand is how to come up with the definitions and how to derive/discover the relevant formulas. The examples we share often are not examples of the use of some important idea, they instead represent the kinds of overt questions that allow you to discover the underlying ideas.

EECS 16AB are structured in a way that tries to help you achieve the required transition in your attitude and ways of thinking. It emphasizes the idea that there are different ways to think and different approaches. Of course, there can be no true learning without sincere effort. And at times, frustration is a natural accompaniment to effort. You’ve experienced this in video games, sports, the arts, and everything else — this is no different. At times in EECS 16AB, you might find yourself thinking “This problem is unfair! The lecture and discussion did not cover this type of problem!”

If you feel this way, take a deep breath and try to remember what the real goal of your learning is. It is alright if you do not know how to solve a problem right away. You just need to be able to read carefully, start working, and then be able to keep going, even if you make mistakes along the way and have to backtrack. After you succeed in doing this many times, overcoming your frustration each time, approaching the unknown will get less scary over time. There is no shortcut to achieving true confidence.

If you succeed in internalizing the “how to think” lessons in EECS 16AB, we hope they will help you tremendously in all your future endeavors. The specific technical skills and concepts taught here are also incredibly fertile and prolific, and you will only begin to see glimpses of their power here.

0.1.2 How you can succeed in this course

The short answer for how to succeed is don’t fall behind, and catch up if you ever do. Attend lecture, pay attention and actively take your own notes, and participate in discussion. Read the official course notes and strive to actually understand what they are explaining to you. Ask questions if things are unclear even after you have thought about it and tried to figure it out. Accept the fact that you might be confused when you first encounter any given HW problem — but don’t let that stop you from trying. Do not be afraid to try something that doesn’t succeed at first. Overcoming what you do not understand and your own fears is a big part of the learning process. Work with others and ask for help from course staff. This is a course in which truly understanding all the labs, homeworks, and discussions is the key to success on exams and beyond.

0.2 System design

Returning to the material of the course. Let’s remember that as engineers, we want to solve meaningful problems. Usually, this requires building a system that can sense something about the real world, compute something useful, and then communicate relevant information or act on this information to change the environment. In 16A, we will mostly focus on the first two problems — sensing and computing — in the context of three applications that you’ll investigate in lab: imaging, touch sensing, and positioning. 16B builds on the knowledge developed in 16A to explore how we can use the information we compute to interact with the environment.

In general, information devices and systems require (1) a real-world quantity to measure, (2) a means of translating this quantity to the electrical domain, and (3) some computation to extract information from the electrical readings. We can represent this flow graphically for the three applications studied in this course:

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4Why do we care about electrical signals specifically? First, pretty much any physical quantity that we care about can be translated into a measurable electrical quantity. Second, most processing techniques can be implemented efficiently with computers,
In an imaging system, our goal is to get a picture of something physical. In lab, you’ll be using a component called a photodiode to convert visible light into a measurable electrical quantity known as current. Modern cameras have millions of these photodiodes — one for each image pixel — but in lab, we’ll explore how to use linear algebra techniques to build an image with only one photodiode.

The goal of a touchscreen is to determine the position of one or more fingers from a user. In the second module of this class, you’ll be exploring a few different ways that touchscreens can be built. In one approach, you will use a circuit called a resistive voltage divider to translate the position of a touch into a voltage. In the second approach, we’ll use the fact that the presence or absence of human touch can influence an electrical quantity called capacitance, which is a principle used in most touchscreens that you interact with today.

In a positioning system, the goal is to find your location in a broader physical space. In lab, we will use a microphone to detect sounds from three speakers at fixed locations — translating information to the electrical domain — and then use optimization techniques to compute the microphone’s location from these readings. In real-world GPS systems, these speakers are replaced by satellites at orbiting the earth at known positions.

0.3 Dealing with complexity

Modern information devices and systems often integrate many sub-systems which are quite complex even independently. For instance, a single smartphone typically has all three of the sub-systems we will study in this class — a camera, touchscreen, and GPS system — in addition to many other features (perhaps a fingerprint reader, voice control, etc.). While the focus of 16A will be on understanding the basics of these sub-systems, approaching even these simpler problems requires multi-step thinking.

In previous classes, you may have had homeworks that involved only plugging numbers into an equation to find your answer. In this class, problems will typically require multiple steps — finding an appropriate model for a real-world system, representing this with the right equations, solving this, interpreting the solution, and perhaps using the results to do even more analysis. Our motivation for doing this is to give you practice which operate on electrical signals. Today (and in the foreseeable future) this computing is done with semiconductors, and thanks to “Moore’s law” — the exponential scaling of semiconductor feature sizes over time — more and more computational processing power has become available via integrated circuits.
thinking in a way that can be applied to complex real-world problems. It may be difficult, but in the end you’ll learn from it, and the course staff is here to help you along the way.

0.4 From models to design

The first part of 16A focuses on building the conceptual tools of linear algebra to start modeling the real world. The second module introduces the idea of circuits to take the next step — going from models to design. There is a difference between analysis of a system (what happens) to design thinking (what do I build so that I can have a system that does what I want) and the second part of 16A will go into this in much more detail. Here the abstraction we use is that of electrical circuits — the nice thing is that they are easily modeled by simple linear equations (that we can solve) and they are fundamental to our ability to build computational devices. Building a full machine learning pipeline requires us to be able to take data from the real world, analyze it, model the world, create designs that can have the impact we want, and then implement this. Through the 16A-16B series we hope to give you the opportunity to experience this full stack through the cumulative experience of the labs. Each of the labs gives you this in small part but the two classes together really help put this together. Understanding how basic circuits works enables this.

The key example we talk about is the touchscreen, but there are other examples too. Here are some more reasons why circuit design is an essential part of this course.

• **Applications**: Can you imagine how useful a smartphone would be without a touchscreen? What about finding your way without GPS tracking technology? Today, we don’t think twice about being able to talk to someone halfway across the earth in real-time, but none of these things would be possible without innovations in circuits. Beyond the fact that all computer processing runs on integrated circuits, circuits shape the way we interact with electronic devices to help give them value — to actually interface with the real world, any device is going to require circuits. Moreover, it is difficult to truly create something new without a full understanding of how an actual engineering system is designed at both the hardware and software level. For instance, some companies like Apple practice “vertical integration,” where engineers work on nearly all aspects of a system, from the mechanical design of a product to the software down to the electrical hardware implementation. Without at least a foundational understanding of circuits, it would be difficult to create an innovative system, which is becoming increasingly relevant in industry with the growth of “internet of things” (IoT) based devices in recent years.

• **Understanding design primitives**: Building any type of complex system — be it software, mechanical, electrical, something else, or any combination of these — requires breaking it into a set of simpler sub-systems, which might consist of even more sub-systems. Introducing different layers of abstraction in this manner is an important tool for managing complexity, and circuit design is one of the most powerful ways of introducing you to this process. Just as software programs are typically built from many subroutines constructed from a set of primitive data types (strings, integers, floating-point numbers, lists, etc.), an electrical circuit typically consists of sub-circuits built from simple primitive building blocks (resistors, capacitors, voltage/current sources) that place constraints on each other.

• **Translation to other fields**: Most circuits can be expressed as a system of equations that can be solved using linear algebraic techniques, making circuit analysis just one example of how the linear algebra concepts you’ll learn about in this class can be used. In fact, this mathematical approach to circuit analysis ties directly to other fields of engineering, which can generally be characterized as
the study of “effort” and “flow” variables — one variable that applies a force (“effort”), and another variable that moves in response to this (“flow”). What does that mean? In electrical circuit analysis, the “effort” variable is voltage, and the “flow” variable is current. In mechanical engineering, the “effort” variable could be a physical force on an object, and the corresponding “flow” variable would be the velocity of this object. In chemical engineering, “effort” is a chemical potential, while “flow” is molar flow, the amount of a substance that passes into a fixed area over time. While you’ll be learning about these ideas in the context of circuits specifically in 16A, they are relevant to many other applications. Even circuit components like resistors and capacitors have mechanical analogues — resistance behaves like a mechanical damping force such as friction, while capacitance is like the compliance of a spring.⁵

- **Relevance to neural networks:** Because circuit analysis translates to a wide range of fields, we can model many physical systems as electrical circuits, often gaining insight about the system. You may have heard of neural networks, an important machine learning tool that can be used to “learn” tasks such as image and voice recognition from examples instead of explicit programming. Neural networks are modeled after biological neural networks, which are fundamentally circuits operating on electrical signals within a brain:

In a general sense, studying circuits provides you with the conceptual and mathematical tools needed to analyze such networks. More broadly, circuit concepts are relevant to understanding network analysis and signal flows in systems, which can be applied to areas ranging from transportation analysis to social network analysis.

- **Interdisciplinary thinking:** Our last example with neural networks brings up an important point: being exposed to a broad range of disciplines can help you come up with new ideas. In the case of neural networks, biological systems inspired a key modern machine learning tool. Understanding concepts outside of your field gives you a broader range of ideas to pull from when trying to develop innovative solutions.

⁵There’s a whole Wikipedia article on the matter if you want more details: [https://en.wikipedia.org/wiki/Mechanical-electrical_analogies](https://en.wikipedia.org/wiki/Mechanical-electrical_analogies)
0.5 Additional Resources

Throughout these notes you will find references to additional resources on the topics covered in this class. The following two textbooks will be referred to in the “Additional Resources” boxes in some of the upcoming notes.

- **Introduction to Linear Algebra by Gilbert Strang, 5th Ed.** (referred to as *Strang* in these notes)
- **Linear Algebra by Lipschutz, Seymour and Lipson, Marc, Schaum’s Outlines, 5th Ed.** (referred to as *Schaum’s* in these notes)
Acknowledgements

The reality is that these notes are the combined effort of many people. In particular, credit for the overall vision of the Linear-Algebraic side of the course and how to present these ideas belongs largely to Gireeja Ranade. The overall aesthetic of 16A owes a great deal to both Gireeja Ranade and Elad Alon, the main instructors for the pilot Sp15 offering and designers of the 16AB course sequence.

The students in the Sp15 pilot offering took detailed lecture notes and wrote them in LaTeX and those formed the basis for much of these official course notes (that continue to evolve). The other instructors in the pilot, namely Babak Ayazifar, Claire Tomlin, and Vivek Subramaniam also had considerable influence on the ideas here. Furthermore, the faculty who helped put together the ideas for the labs, like Miki Lustig and Laura Waller (imaging) and Tom Courtade (locationing), and Michel Maharbiz (spirit), helped shape the overall cadence of the course that these readings are meant to complement. The undergrads who worked tirelessly over the summer (2015) after the pilot shaped these notes considerably, with particular credit going to Chenyang Yuan, Rachel Zhang, and Siddharth Karamcheti.

Much work was put into the notes over the years. In 2015 there was a lot of input from Chenyang Yuan and Anant Sahai. The other members of the TA group that semester have also contributed. In Sp16, Jennifer Shih worked closely with Anant Sahai and the then 16A instructors Elad Alon and Babak Ayazifar to get these notes actually released. In Su17, Dominic Labanowski, Sajjad Moazeni, Nick Sutardja, Angiie Wang, and Amy Whitcombe worked with Elad Alon and Anant Sahai to further develop the 16AB course material. In following semester Vladimir Stojanovic and Grace Kuo worked on the notes as well as tremendous inputs from Rahul Arya and Gireeja Ranade. A major new and radically different freshman course doesn’t just happen — lots of people in the EECS department worked together for years to make all this a reality.

Proper credit here also acknowledges the strong cultural influence of the faculty team that made EECS 70 and shaped it into a required course: Alistair Sinclair, Satish Rao, David Tse, Umesh Vazirani, Christos Papadimitriou, David Wagner, and Anant Sahai, along with pioneering TA’s like Thomas Vidick, all helped make and polish the notes for 70. The 16AB+70 sequence all follows the aesthetic that mathematical ideas should have practical punchlines. 16AB emphasizes the rooting of mathematical intuition in physical examples as well as virtual ones.