Lecture 4C: (7/12/23)

Announcements:
- Module #2 truly begins!
- Office Hours - Mon/Wed 1-2pm in Cory 144MA
  - Today!
- Lab - Please consider moving from afternoon to evening section

- Today's Topics: How to "solve" a circuit
  - Review of circuit concepts (Note 11A)
  - Kirchhoff's Laws (Note 11A)
  - Node Voltage Analysis (Note 11B)

Node Voltage Analysis

Solve this circuit.

\[ V_s = 10V \]
\[ R_1 = 15\, \Omega \]
\[ R_2 = 25\, \Omega \]
\[ R_3 = 25\, \Omega \]

What is a node? How many nodes? 3
What is \( V_s \)?
What is \( R \)?
Ohm's Law: \( V = IR \)

How do we "solve"?
What is current?

\[ i = \frac{dq}{dt} \quad \text{current is flow of charge} \]

What is charge?

\[ q \quad \text{C (coulomb)} \]

- electrons!  
  - charge of 1 electron is \[ e = -1.602 \times 10^{-19} \text{C} \]
  - 1 coulomb is \[ 6.243 \times 10^{18} \text{electrons} \]

What is voltage?

\[ V \quad \text{V (volts)} \]

- can make charge move
  \[ E = q \cdot V \]
  - potential energy

- the amount of energy it takes to move a charge to a new voltage
- voltage is relative (How high is my hand?)

Nodo:

A point where two or more circuit elements meet.
A region that is equipotential (equal voltage potential throughout)
Kirchhoff’s Current Law: “KCL”

1. Sum of currents entering and exiting a node is zero

\[ +I_1 - I_2 - I_3 = 0 \]

2. Sum of entering currents = sum of exiting currents

\[ I_1 = I_2 + I_3 \]

- Both definitions are equivalent

Loop:
A closed path in a circuit. Ends where it starts.

Kirchhoff’s Voltage Law: “KVL”

1. Sum of voltages in any loop is zero

\[ +V_S - V_1 - V_2 = 0 \]

2. Sum of voltage rises = sum of voltage drops

\[ V_S = V_1 + V_2 \]
Hiking Example:

Let's solve the circuit using "Node Voltage Analysis."

"Write a KCL equation at every node in terms of node voltages and IV characteristics." ~ Nathan

\[ V_s = 10 \text{V} \]
Step 1: Pick a reference node ("ground node") \( V_s \) 
- can pick ANY node

Step 2: Label all remaining nodes "\( V_i \)"
- 2 nodes \( V_1 \) and \( V_2 \) (exclude reference node remaining)

Step 3: Label the current through every element "\( i_i \)"
- Does \( i_a = i_6 \)?
  - KCL: \( i_a = i_6 \)

Step 4: Add voltage labels (+/-) across each element

Passive sign convention:
- current enters positive terminal and exits negative terminal

\[ + \quad \text{V} \quad - \]

\[ \text{I} \]
Step 5: Identify all unknowns. Simplify if possible.

Node voltages: \( U_1, U_2 \)

Element currents: \( i_0, i_1, i_2, i_3 \)

Element voltages: \( V_x, V_1, V_2, V_3 \)

These are the important ones.

Step 6a: Set up a system of KCL equations.

At node \( U_1 \): \( i_5 - i_1 = 0 \)
At node \( U_2 \): \( i_1 - i_2 - i_3 = 0 \)

Step 6b: Use I-V relationships (e.g., Ohm’s Law)

Element voltages \( \rightarrow \) node voltages \( \Rightarrow \) \( \left\{ \begin{array}{l}
V_x = 0 - U_1 \\
V_1 = U_1 - U_2 \\
V_2 = U_2 - 0 \\
V_3 = U_2 - 0
\end{array} \right\} \Rightarrow \left\{ \begin{array}{l}
u_1 = U_5 \\
u_1 - u_2 = i_1 R_1 \\
u_2 = i_2 R_2 \\
u_3 = i_3 R_3
\end{array} \right\}

We’ve eliminated “element voltage” variables!

Step 7: Simplify equations and solve

KCL equations:
\[
\begin{align*}
i_5 - i_1 &= 0 \\
i_1 - i_2 - i_2 &= 0
\end{align*}
\]

I-V equations:
\[
\begin{align*}
u_1 &= U_5 \\
u_1 - u_2 &= i_1 R_1 \\
u_2 &= i_2 R_2 \\
u_3 &= u_2
\end{align*}
\]
\[ \begin{align*}
V_5 - V_1 &= 0 \\
i_1 - i_2 - i_3 &= 0
\end{align*} \]

\[ i_5 = \frac{V_5 - V_2}{R_1} \]

\[ \begin{align*}
V_1 - V_2 &= i_1 R_1 \\
V_2 &= i_2 R_2 \\
V_2 &= i_3 R_3
\end{align*} \]

\[ i_2 = \frac{V_2}{R_2} \]

\[ i_3 = \frac{V_2}{R_3} \]

1. \[ i_5 = \frac{V_5 - V_2}{R_1} = 0 \]

   Eliminated element currents!

2. \[ \frac{V_1 - V_2}{R_1} - \frac{V_2}{R_2} - \frac{V_2}{R_3} = 0 \]

3. \[ V_1 = V_5 \]

Matrix vector form: \[ \mathbf{A} \mathbf{x} = \mathbf{b} \]

1. \[ \begin{bmatrix}
-\frac{1}{R_1} & \frac{1}{R_1} & 0 \\
\frac{1}{R_1} & -\left(\frac{1}{R_2} + \frac{1}{R_3}\right) & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2 \\
i_5
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
V_5
\end{bmatrix} \]

This method ALWAYS works. Let's simplify though.

3. \[ \frac{V_5 - V_2}{R_1} - \frac{V_2}{R_2} - \frac{V_2}{R_3} = 0 \]

\[ V_5 = 10V \]
\[ R_1 = 1\Omega \]
\[ R_2 = 2.5\Omega \]
\[ R_3 = 2.5\Omega \]

\[ V_5 = \frac{1}{R_1 + \frac{1}{R_2} + \frac{1}{R_3}} \cdot V_2 \]

\[ V_2 = \left(\frac{1}{R_1 + \frac{1}{R_2} + \frac{1}{R_3}}\right)^{-1} \cdot \frac{1}{R_1} \cdot V_5 \]

\[ V_2 = \left(\frac{1}{R_1 + \frac{1}{R_2} + \frac{1}{R_3}}\right)^{-1} \cdot \frac{1}{R_1} \cdot 10 \]

\[ \approx 9 \rangle^\circ 10 \]
We've solved the circuit!

Check Results:

\[ U = iR \rightarrow i = \frac{U}{R} \]

Check KCL:

\@ u_2: 5A = 2.5A + 2.5A

Check KVL

\[ +10V - 5V - 5V = 0V \]