1. HONOR CODE

Please read the following statements of the honor code, and sign your name (you don’t need to copy it).

_I will respect my classmates and the integrity of this exam by following this honor code. I affirm:_

- I have read the instructions for this exam. I understand them and will follow them.
- All of the work submitted here is my original work.
- I did not reference any sources other than my one reference cheat sheet.
- I did not collaborate with any other human being on this exam.

2. Tell us a joke or pun about circuits (1 point) All answers will be awarded full credit.
3. Why is the ammeter so hot? (14 points)

In the following circuit, we would like to measure the current $I_m$ through resistor $R_L$ using an ammeter.

(a) (4 points) If $I_S = 4 \text{ A}$, $R_S = 3 \text{ m}\Omega$, and $R_L = 1 \text{ m}\Omega$, then what is the measured current $I_m$? Assume an ideal ammeter.

**Solution:**

The ideal ammeter is an equivalent short circuit.

Using the current divider equation, we have

$$I_m = \frac{R_S}{R_S + R_L} \cdot I_S = \frac{(3 \text{ m}\Omega)}{(3 \text{ m}\Omega) + (1 \text{ m}\Omega)} (4 \text{ A})$$

$$I_m = 3 \text{ A}.$$  

(b) (4 points) Now suppose we have different values of $I_S$ and $R_S$ such that $I_m = 2 \text{ A}$, then what is the power dissipated by $R_L = 1 \text{ m}\Omega$? How much energy does $R_L$ dissipate in 20 seconds?

**Solution:** To find the energy dissipated by $R_L$ we can consider the current $I_L$ through it and then use one of the power equations for a resistor $P = I^2 R = R$. Doing this yields

$$I_L = I_m = 2 \text{ A}$$

$$P_L = I_L^2 R_L = (2 \text{ A})^2 (1 \text{ m}\Omega) = 4 \text{ mW}$$

The amount of energy dissipated by $R_L$ in 20 seconds is just the product of power and time (since power is constant) which gives

$$E_L = P_L \Delta t = (4 \text{ mW})(20 \text{ s})$$

$$= 80 \text{ mJ}$$
(c) (6 points) Now assume the ammeter is not ideal and can instead be modeled with equivalent resistance $R_A$. That is, we can replace the ammeter with a resistor $R_A$:

If $I_S = 4 \text{ A}$, $R_S = 3 \text{ m}\Omega$, $R_L = 1 \text{ m}\Omega$, and the current measured by the ammeter (i.e., the current through $R_A$ ) is $I_m = 2 \text{ A}$, then what must be the internal resistance $R_A$ of the ammeter? How much power is dissipated by the ammeter?

Solution:
The equivalent circuit replaces the non-ideal ammeter with an equivalent resistance $R_A$.

Using a current divider and combining $R_A$ and $R_L$ we get,

$$I_m = \frac{R_S}{R_S + (R_A + R_L)} \cdot I_S$$

$$I_m R_A = I_S R_S - I_m \cdot (R_S + R_L)$$

$$R_A = \frac{I_S}{I_m} R_S - (R_S + R_L)$$

$$= \frac{(4 \text{ A})}{(2 \text{ A})} (3 \text{ m}\Omega) - (3 \text{ m}\Omega + 1 \text{ m}\Omega)$$

$$R_A = 2 \text{ m}\Omega$$

The power dissipated by the ammeter is then

$$P_A = I_m^2 R_A = (2 \text{ A})^2 (2 \text{ m}\Omega)$$

$$= 8 \text{ mW}$$
4. Just Solvin’ a Circuit (18 points)

(a) (2 points) For this part only, consider the following circuit. For each element in the circuit (including the sources!), label the missing current or voltage using passive sign convention. Please label directly on the given circuit below. Voltage labels should have +/- terminals. For example, the voltage source $V_{s1}$ has its voltage already labeled ($V_5$) but is missing its current label.

![Circuit Diagram]

Solution: Every element should have a voltage, current, and +/- terminals labeled. The current should flow from the positive terminal to the negative terminal following passive sign convention.
Now, for the remaining parts, consider an entirely different circuit which is already labeled.

(b) (2 points) Write the KVL equation for the labeled loop in terms of element voltages \((V_1, V_2, \ldots, V_8)\) only.

**Solution:** Depending on which convention is used,

\[-V_4 - V_5 + V_8 + V_6 = 0\]

or

\[V_4 + V_5 - V_8 - V_6 = 0\]

or any equivalent expression.

(c) (4 points) Write the KCL equations for nodes \(u_1\) and \(u_2\) ONLY in terms of \(I_{s1}, I_{s2}\), node voltages (not element voltages!), and resistances.

**Solution:**

For \(u_1\),

\[i_1 + i_2 + i_7 = 0\]

\[i_1 + i_2 - I_{s1} = 0\]

\[\frac{u_1}{R_1} + \frac{u_1 - u_3}{R_2} - I_{s1} = 0\]

or equivalent equations

For \(u_2\),

\[i_3 + i_4 + i_7 = i_5\]

\[\frac{u_3 - u_2}{R_3} - \frac{u_2}{R_4} - I_{s1} + I_{s2} = 0\]
or equivalent equations. Note that $I_{s2}$ could also be replaced with $\frac{\mu_2 - \mu_4}{R_5}$ or $-\frac{\mu_5}{R_6}$. 
Here the circuit is redrawn for your convenience:

(d) (10 points) Solve for the voltage $V_1$ across and current $I_1$ through $R_1$ given the following values:

$R_1 = 4\Omega$, $R_2 = 2\Omega$, $R_3 = 1\Omega$, $R_4 = 1\Omega$, $R_5 = 7\Omega$, $R_6 = 4\Omega$

$V_s = 6V$, $I_{s1} = 9A$, $I_{s2} = 3A$

You do NOT need to solve for all quantities in the circuit nor use all of the above given values in your answer.
Here the circuit is redrawn for your convenience:

Solution: With these given values, we have the following circuit:

To find $i_1$ and $V_1$, we perform superposition on the circuit.

First, turn $V_s$ on and zero out all other independent sources. This means that $I_{s1}$ and $I_{s2}$ behave like open circuits.
By inspection, the left half of the circuit is a voltage divider. By the voltage divider formula,

\[ V_1 = \frac{4\Omega}{2\Omega + 4\Omega} \cdot 6V = \frac{2}{3} \cdot 6V = 4V \]

Using Ohm’s Law, we can find \( i_1 \) from \( V_1 \):

\[ i_1 = \frac{V_1}{R_1} = \frac{4V}{4\Omega} = 1A \]

Next, turn on \( I_{s1} \) and zero out all other independent sources. This means that \( V_s \) behaves like a wire and \( I_{s2} \) behaves like an open circuit.

By inspection, we have a current divider, where the 9A current splits into the 4\( \Omega \) and 2\( \Omega \) branches. We can then apply the current divider formula as follows:

\[ i_1 = \frac{2\Omega}{2\Omega + 4\Omega} \cdot 9A = \frac{1}{3} \cdot 9A = 3A \]
Using Ohm’s Law, we can find $V_1$ from $i_1$:

$$V_1 = i_1 R_1 = 3A \cdot 4\Omega = 12V$$

Alternatively, we can solve this via NVA:

We write KCL for node $u_1$ and solve:

$$\frac{u_1}{R_1} + \frac{u_1}{R_2} = I_{s1}$$

$$\frac{R_2u_1 + R_1u_1}{R_1R_2} = \frac{(R_1 + R_2)u_1}{R_1R_2} = I_{s1}$$

$$V_1 = u_1 = \frac{R_1R_2}{R_1 + R_2} I_{s1} = \frac{8\Omega^2}{6\Omega} \cdot 9A = 12V$$

$$i_1 = \frac{V_1}{R_1} = \frac{R_2}{R_1 + R_2} I_{s1} = \frac{2\Omega}{4\Omega + 2\Omega} \cdot 9A = 3A$$

Lastly, turn on $I_{s2}$ and zero out all other independent sources. This means that $V_s$ behaves like a wire and $I_{s1}$ behaves like an open circuit.
We can simplify the circuit by combining resistors $R_1$ and $R_2$ which are in series:

By inspection, the $R_1 + R_2$ resistor is shorted because it is connected in parallel to a wire. This means there is no current flowing through the $R_1 + R_2$ resistor. To see this, we can use Ohm’s Law:

$$u_3 = \text{ground} = 0$$
$$i_1 = \frac{u_3}{R_1 + R_2} = \frac{0}{R_1 + R_2} = 0 \text{A}$$
$$V_1 = i_1 R_1 = 0 \text{V}$$
Alternatively, we can solve this via NVA by using KCL at node $u_1$:

\[
\begin{align*}
    i_1 &= i_2 \\
    \frac{u_1}{R_1} &= \frac{u_1}{R_2}
\end{align*}
\]

Since $R_1 \neq R_2$, the above equation is only true if $u_1 = 0$. Then,

\[
\begin{align*}
    i_1 &= \frac{u_1}{R_1} = 0A \\
    V_1 &= i_1R_1 = 0V
\end{align*}
\]

Finally, using superposition we can sum up the contributions from all three sources to find $i_1$ and $V_1$ of the complete circuit:

\[
\begin{align*}
    i_1 &= i_1\bigg|_{V_{s,1}} + i_1\bigg|_{I_{s,1}} + i_1\bigg|_{I_{s,2}} = 1A + 3A + 0A = 4A \\
    V_1 &= V_1\bigg|_{V_{s,1}} + V_1\bigg|_{I_{s,1}} + V_1\bigg|_{I_{s,2}} = 4V + 12V + 0V = 16V
\end{align*}
\]
5. Op Amps and Comparators (17 points)

(a) (2 point) Consider the following op-amp for this part only. Notice the + and - signs are missing on the op-amp input. Please label them such that the op-amp is in negative feedback.

Solution:

This circuit is in negative feedback because "dinking" the output voltage of the Op-Amp causes $U^-$ (the negative input terminal of the Op-Amp) to increase as well which then causes $V_{out} = A(U^+ - U^-)$ to decrease. Since $U^-$ is in some way connected back to $V_{out}$ and the dinked output led to a subsequent change in the opposite direction, the circuit is indeed in negative feedback.
For the next two parts consider an entirely new op-amp circuit. Determine the following (you can assume that the op-amp is ideal and in negative feedback):

(b) \( R_2 \)
\( R_1 \)
\( V_{in} \)
\( R_3 \)
\( V_{out,1} \)

(b) (3 point) What is the voltage held at the negative terminal of the op-amp (the node labeled (b))? Express your answer in terms of the variables shown in the circuit diagram.

Solution:
Since it is given that we are working with an ideal Op-Amp and our Op-Amp is in negative feedback, we can apply the second golden rule to this circuit \( (U^+ = U^-) \). Since \( U^+ \) is connected to ground node and we know \( (U^+ = U^-), U^- \) must be 0V.

(c) (6 points) What is \( I_{out} \), the current flowing through \( R_L \)? Express your answer ONLY in terms of \( V_{in}, R_1, R_2, R_3, R_L \). You cannot use \( V_{out,1} \) in your final expression.

Solution:

The above circuit uses ideal Op-Amps and is in negative feedback. Thus, we can apply the two golden rules:
1. \( I^+ = I^- = 0A \)
2. \( U^+ = U^- = 0V \) (from part b)

We can solve for \( V_{out,1} \) first and then use Ohm’s law to get the desired \( I_{out} \).

To begin, we perform KCL on \( U^- \):
\[ I_{R_1} + I_{R_3} = I_{R_2} \]
\[ V_{R_1} + V_{R_3} = U^- \]
\[ \frac{V_{R_1}}{R_1} + \frac{V_{R_3}}{R_3} = \frac{U^-}{R_2} \]
\[ \frac{V_{out,1}}{R_3} = -\frac{V_{in}}{R_1} \]

Solving for \( V_{out,1} \), we get:

\[ V_{out,1} = -\frac{R_3}{R_1}V_{in} \]

Finally, applying Ohm’s law gives

\[ I_{out} = \frac{V_{out,1}}{R_L} = -\frac{R_3}{R_1R_L}V_{in} \]
(d) (3 points)
Now consider attaching an ideal comparator to the output of the op-amp circuit from the previous part.

Suppose the output voltage of the op-amp is $V_{out,1} = -2V$. What is $V_{out,2}$, the voltage outputted by the comparator?

Solution:

We know that when the input voltage at the negative terminal of the comparator exceeds that at the positive terminal, then we output $V_{SS}$. However, when the input voltage at the positive terminal of the comparator exceeds that at the negative terminal, then we output $V_{DD}$. When they are equivalent to each other (i.e. $U_c^+ = U_c^-$), then we see the comparator instantaneously change from outputting $V_{SS}$ to outputting $V_{DD}$. Therefore, we can compute the difference of the voltages at the two input terminals to determine the output of the comparator:

$U_c^+ = V_{out,1} = -2V$

$U_c^- = 3V$

$U_c^+ - U_c^- = -2V - 3V = -5V < 0$. Therefore, $V_{out,2} = V_{SS} = -5V$

(e) (3 points) Sketch the output behavior of the comparator on the plot provided for different values of $V_{out,1}$. 

EECS 16A, Fall 2022, Midterm 2
Solution:

Observe that we always output the value of $V_{SS}$ when $U_c^+ < U_c^-$, switch from outputting $V_{SS}$ to outputting $V_{DD}$ when $U_c^+ = U_c^-$, and remain outputting $V_{DD}$ for the entire time that $U_c^+ > U_c^-$. Also observe that the shape of the graph takes on the shape of the internal labeling of the comparator!
6. Oski is Soldering a Resistive Touchscreen (12 points)

(a) (4 points) Oski attended Touchscreen 2 lab and was able to make a resistive touchscreen model that supports 9 touches. The schematic of the touchscreen bottom mesh is shown on Fig. 1.

Knowing that the lab model suggests to take all 12 resistors to have the same value, and $V_s = 8V$, what are the voltage values on each node? Fill out the table:

<table>
<thead>
<tr>
<th>node name</th>
<th>voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>u1</td>
<td></td>
</tr>
<tr>
<td>u2</td>
<td></td>
</tr>
<tr>
<td>u3</td>
<td></td>
</tr>
<tr>
<td>u4</td>
<td></td>
</tr>
<tr>
<td>u5</td>
<td></td>
</tr>
<tr>
<td>u6</td>
<td></td>
</tr>
<tr>
<td>u7</td>
<td></td>
</tr>
<tr>
<td>u8</td>
<td></td>
</tr>
<tr>
<td>u9</td>
<td></td>
</tr>
</tbody>
</table>

Solution:
<table>
<thead>
<tr>
<th>node name</th>
<th>voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>u1</td>
<td>6</td>
</tr>
<tr>
<td>u2</td>
<td>8</td>
</tr>
<tr>
<td>u3</td>
<td>6</td>
</tr>
<tr>
<td>u4</td>
<td>4</td>
</tr>
<tr>
<td>u5</td>
<td>4</td>
</tr>
<tr>
<td>u6</td>
<td>4</td>
</tr>
<tr>
<td>u7</td>
<td>2</td>
</tr>
<tr>
<td>u8</td>
<td>0</td>
</tr>
<tr>
<td>u9</td>
<td>2</td>
</tr>
</tbody>
</table>

\[
u_8 = 0V \text{ (ground node)}
\]

\[
u_2 = 8V \text{ (from the voltage source)}
\]

\[
u_5 = \frac{1}{2}u_2 = 4V
\]

\[
u_4 = u_6 = u_5 = 4V
\]

\[
u_1 = u_3 = 4V + \frac{1}{2}(8 - 4)V = 6V
\]

\[
u_7 = u_9 = \frac{1}{2}(4 - 0)V = 2V
\]
(b) (2 points) The lab Oski is in only has a two different types of resistor, $R_1$ and $R_2$. They are both 4-band resistors but have different colored bands: $R_1$ has yellow-violet-red-gold and $R_2$ has red-violet-yellow-gold. What are the values of resistances $R_1$ (yellow-violet-red-gold) and $R_2$ (red-violet-yellow-gold). Include the tolerance value in your answer (you may leave it as a percentage). You may put your answer in scientific notation. Refer to Fig. 2 for the color code convention.

<table>
<thead>
<tr>
<th>Color</th>
<th>1st Band</th>
<th>2nd Band</th>
<th>3rd Band Multiplier</th>
<th>4th Band Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>0</td>
<td>0</td>
<td>x1Ω</td>
<td>±1%</td>
</tr>
<tr>
<td>Brown</td>
<td>1</td>
<td>1</td>
<td>x10Ω</td>
<td>±2%</td>
</tr>
<tr>
<td>Red</td>
<td>2</td>
<td>2</td>
<td>x100Ω</td>
<td>±5%</td>
</tr>
<tr>
<td>Orange</td>
<td>3</td>
<td>3</td>
<td>x1kΩ</td>
<td>±0.5%</td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
<td>4</td>
<td>x10kΩ</td>
<td>±0.25%</td>
</tr>
<tr>
<td>Green</td>
<td>5</td>
<td>5</td>
<td>x100kΩ</td>
<td>±0.10%</td>
</tr>
<tr>
<td>Blue</td>
<td>6</td>
<td>6</td>
<td>x1MΩ</td>
<td>±0.05%</td>
</tr>
<tr>
<td>Violet</td>
<td>7</td>
<td>7</td>
<td>x10MΩ</td>
<td>±5%</td>
</tr>
<tr>
<td>Grey</td>
<td>8</td>
<td>8</td>
<td>x100MΩ</td>
<td>±10%</td>
</tr>
<tr>
<td>White</td>
<td>9</td>
<td>9</td>
<td>x1GΩ</td>
<td>±1%</td>
</tr>
<tr>
<td>Gold</td>
<td></td>
<td></td>
<td>x0.1Ω</td>
<td>±5%</td>
</tr>
<tr>
<td>Silver</td>
<td></td>
<td></td>
<td>x0.01Ω</td>
<td>±10%</td>
</tr>
</tbody>
</table>

Fig. 2: 4-band resistor color code convention table.

Solution:

\[ R_1 \text{(yellow-violet-red-gold)} = 47 \times 10^2 \pm 5\% = 4.7k\Omega \]
\[ R_2 \text{(red-violet-yellow-gold)} = 27 \times 10^4 \pm 5\% = 270k\Omega \]
(c) (6 points) It turns out that Oski’s bottom mesh does not work as it is proposed in the lab. Looking at the photo of Oski’s circuit in Fig. 3, we see that Oski accidentally used $R_2$ instead of $R_1$ on the right side of the circuit. As a result, not all the resistor values are the same, which was a requirement we saw in part (a). Regardless of the values in the previous parts, assume that $R_1 = 1k\Omega$ and $R_2 = 2k\Omega$ and the voltage source $V_s = 8V$ remains the same as part a). What are the voltages at nodes $u_3$ and $u_6$?

Fig. 3: A buggy bottom mesh of a 2D touchscreen.

Solution: They key thing to note is that the currents between $u_4$ and $u_5$ as well as between $u_5$ and $u_6$ is 0. Thus the desired node voltages can be found using voltage dividers

$$u_6 = \frac{R_1 + R_2}{2 \times (R_1 + R_2)} u_2 = \frac{1}{2} u_2 = 4V$$

$$u_3 = \frac{R_2 + R_2 + R_1}{2 \times (R_1 + R_2)} u_2 = \frac{20}{3} V$$
7. Making circuits equivalent (18 points)

(a) (8 points) Suppose you are given a mysterious circuit element shown below. You are also given the IV curve of the circuit at nodes a and b (also shown below).

Now you would like to model this mysterious circuit two different ways. One with each circuit below:

Find the values $V_{th}$, $R_{th}$, $I_{no}$, and $R_{no}$ such that all 3 circuits (the mystery circuit and the two above) are equivalent. Note the direction of $I_{out}$ in the mysterious circuit!

Solution: We recognize the below circuits as the Thevinin and Norton equivalents of the given mystery circuit. Recall that the Thevinin voltage $V_{th}$ is the x-intercept of the IV curve, the Norton current $I_{no}$ is the negative y-intercept of the IV curve, and $R_{th} = R_{no}$ is the inverse of the slope of the IV curve. Thus, just by reading the plot we can deduce that

$$V_{th} = 4V, \quad I_{no} = 2A, \quad R_{th} = R_{no} = \frac{4}{2} = 2\, \Omega.$$ 

(b) (10 points) Suppose now we are given a new circuit (shown below), but this time we know all the components except one resistor $R$: 

---

EECS 16A, Fall 2022, Midterm 2
We also know that we can model the circuit at nodes a and b using the following equivalent circuit:

Find the resistor value $R$ in the new circuit such that $I_{eq} = 0.25\, \text{A}$.

**Solution:** Connecting terminals a and b in the original circuit, we want to find the current $I_{ab}$ going through it, which equals our $I_{eq}$.

The wire ends up shorting the 5Ω resistor so our equivalent circuit looks like this:
Now we can simply combine $R$ with the $20\Omega$ resistor (they share the same current $I_{ab}$) and use the fact that resistors in parallel share the same voltage

\[ I_{ab} = \frac{10}{R + 20} \quad R = \frac{10 - 20I_{ab}}{I_{ab}} = 20\Omega. \]
The circuit is redrawn here for your convenience:
8. Capacitors in the Wild (20 points)

(a) For the following circuit, find the equivalent capacitance between the specified nodes in terms of the given capacitors, $C_i$. You may use the parallel operator $\parallel$ for simplification (e.g. a valid answer format could be $C_{eq} = C_6 + (C_7 \parallel (C_8 + C_9))$).

\[
\begin{align*}
C_{eq} &= C_2 + C_1 \parallel (C_3 + (C_4 \parallel C_5)) \\
C_{eq} &= C_1 \parallel C_2 + C_3 + C_4 \parallel C_5
\end{align*}
\]

(b) (12 points) In the following circuit, switches labeled $\phi_1$ are closed during phase 1 and open during phase 2; switches labeled $\phi_2$ are closed during phase 2 and open during phase 1. Assume before phase 1 all capacitors are fully discharged. Each phase is long enough that the circuit reaches steady-state. Find $V_{out}$ in phase 2 in terms of $V_s$, $C_1$, $C_2$, $C_3$, and $C_4$. You may use the parallel operator $\parallel$ in your final answer.

The circuit is redrawn here for your convenience.
Solution:
First we draw the equivalent circuit diagrams in both phases. Phase 1:

We note that C1 and C2 are in series for both phases, and C3 and C4 are in parallel for both phases as well. Therefore, as we are looking to compute the value of $V_{\text{out}}$, we can combine C1 and C2, and C3 and C4 and work with their equivalent capacitances to simplify calculations. It is possible to solve without this step, but it is more work. Let:

$$C_{12} = C_1 \parallel C_2 = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{34} = C_3 + C_4$$

We obtain the following equivalent circuit:
In this circuit, $C_{12}$ and $C_{34}$ are in parallel, and the voltage across both capacitors is $V_s$. Therefore, the magnitude of charges during phase 1 on both capacitors are

$$Q_{12} = C_{12}V_s \quad (1)$$
$$Q_{34} = C_{34}V_s \quad (2)$$

Now, during phase 2, the circuit looks like the following with our equivalent capacitors:

First, we have that in phase 2: $V_{\text{out}} = V_{34}^{\phi_2}$.

From KVL, we get:

$$V_s + V_{12}^{\phi_2} - V_{34}^{\phi_2} = 0 \quad (3)$$

Also, for each capacitor we have $Q = CV$:

$$Q_{12} = C_{12}V_{12}^{\phi_2} \quad (4)$$
$$Q_{34} = C_{34}V_{34}^{\phi_2} \quad (5)$$

Finally, we have one floating node, which is connected to both positive capacitor plates. Therefore, conserving the change between both phases we have:

$$+Q_{12} + Q_{34} = +Q_{12}^{\phi_2} + Q_{34}^{\phi_2} \quad (6)$$

Now, we solve!

First, plugging (1), (2), (4) and (5) into (6), we get:

$$V_s \cdot (C_{12} + C_{34}) = C_{12}V_{12}^{\phi_2} + C_{34}V_{34}^{\phi_2} \quad (7)$$

Dividing equation (7) by $C_{12}$ and adding with equation (3), we get:

$$V_s + V_{12}^{\phi_2} + V_s \frac{C_{12} + C_{34}}{C_{12}} = V_{12}^{\phi_2} + C_{34}V_{34}^{\phi_2}$$
Where, we can solve for $C_{34}$ (which is equal to $V_{out}$):

$$V_s \left( 1 + \frac{C_{12} + C_{34}}{C_{12}} \right) = V_{\phi_{34}} \left( 1 + \frac{C_{34}}{C_{12}} \right)$$

$$V_s \left( \frac{2C_{12} + C_{34}}{C_{12}} \right) = V_{\phi_{34}} \left( \frac{C_{12} + C_{34}}{C_{12}} \right)$$

Therefore,

$$V_{\phi_{34}} = V_s \left( \frac{2C_{12} + C_{34}}{C_{12} + C_{34}} \right)$$

Where,

$$C_{12} = C_1 \parallel C_2 = \frac{C_1C_2}{C_1 + C_2}$$

$$C_{34} = C_3 + C_4$$

This answer is enough.

Answer without equivalent capacitances:

$$V_{\phi_{34}} = V_s \left( \frac{2(C_1 \parallel C_2) + C_3 + C_4}{C_1 \parallel C_2 + C_3 + C_4} \right)$$

$$V_{\phi_{34}} = V_s \left( \frac{2 \frac{C_1C_2}{C_1 + C_2} + C_3 + C_4}{\frac{C_1C_2}{C_1 + C_2} + C_3 + C_4} \right)$$

$$V_{\phi_{34}} = V_s \left( \frac{2C_1C_2 + C_1C_3 + C_1C_4 + C_2C_3 + C_2C_4}{C_1C_2 + C_1C_3 + C_1C_4 + C_2C_3 + C_2C_4} \right)$$