

EECS 16A
Fall 2023

Designing Information Devices and Systems I

Final Exam

Exam Location: Dwinelle 155

PRINT your student ID: _____

PRINT your name: _____, _____
(last name) (first name)

PRINT your discussion section and GSI: _____

Name and SID of the person to your left: _____

Name and SID of the person to your right: _____

Name and SID of the person in front of you: _____

Name and SID of the person behind you: _____

0. Honor Code (0 Points)

Acknowledge that you have read and agree to the following statement and sign your name below:
As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others. I will follow the rules and do this exam on my own.

If you do not sign your name, you will get a 0 on the exam.

Empty rectangular box for signature.

1. When the exam starts, write your SID at the top of every page. (3 Points)

No extra time will be given for this task.

2. Tell us about something you are proud of this semester. (2 Points)

Empty rectangular box for answer.

3. What are you looking forward to over winter break? (2 Points)

Empty rectangular box for answer.

Do not turn this page until the proctor tells you to do so. You may work on the questions above.

PRINT your student ID: _____

Extra page for scratchwork.

If you want any work on this page to be graded, explicitly tell us to grade this page number in the original problem's solution area.

PRINT your student ID: _____

4. Strike a Chord (4 points)

Alex built a bot that helps you learn to play the guitar. It listens to you play a melody and compares it to a target melody. Each melody maps to a vector. The target melody you are learning maps to $[1 \ -1 \ 1 \ -1]^T$. You play “melody A” that maps to $[-1 \ 1 \ 1 \ -1]^T$ and “melody B” that maps to $[1 \ -1 \ -1 \ -1]^T$.

A smaller angle between two melodies means they are closer. **Does “melody A” or “melody B” have a smaller angle with the target melody?** Justify your response.

| | | | | | |
|----------------|-----------|----------------------|----------------------|---------------|------------|
| θ | 0° | 30° | 45° | 60° | 90° |
| $\cos(\theta)$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |

Table 4.1: Helpful cosine values.

PRINT your student ID: _____

5. Landing Gear (13 points)

Youbin, a forgetful space pilot, often forgets to deploy his landing gear on his spaceship. Using his knowledge from the capacitive touchscreen lab, he wants to design a circuit that will sense when the spaceship is close to the surface and automatically deploy the landing gear.

- (a) (5 points) Youbin installs two electrodes E_1 and E_2 on the bottom of his spaceship as shown in Figure 5.1.

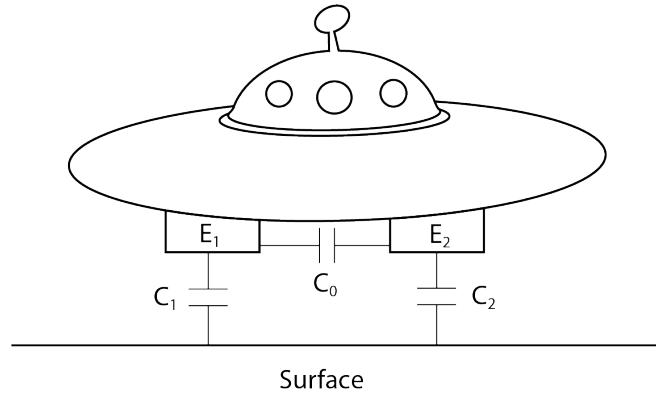


Figure 5.1: Capacitance diagram when landing

The two electrodes form a capacitor with capacitance C_0 . When the spaceship nears the surface, the electrodes also form a capacitor C_1 and C_2 with the surface. The surface can be assumed to be conductive. **Draw a circuit diagram that represents the system when the spaceship is near the surface. Explicitly label the capacitors C_0, C_1, C_2 and the nodes E_1, E_2 and Surface. What is the equivalent capacitance C_{eq} between E_1 and E_2 when the spaceship is near the surface?** You may use the parallel operator in your answer.

PRINT your student ID: _____

- (b) (4 points) In order to detect the change in capacitance, Youbin connects a time-varying current source $I_s(t)$ to the electrodes with effective capacitance C_{eq} , as shown in Figure 5.2.

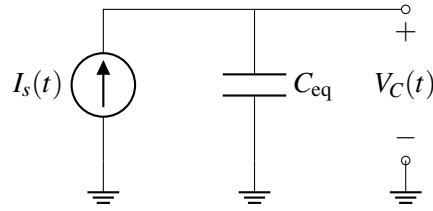


Figure 5.2

He knows that when landing, $C_{eq} = 1 \mu\text{F}$ and $I_s(t)$ outputs a square wave shown in Figure 5.3.

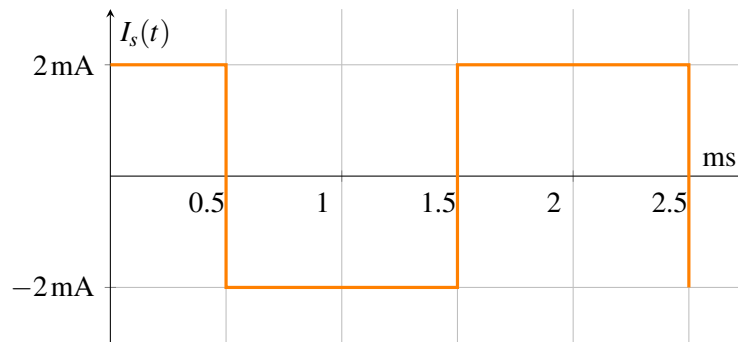
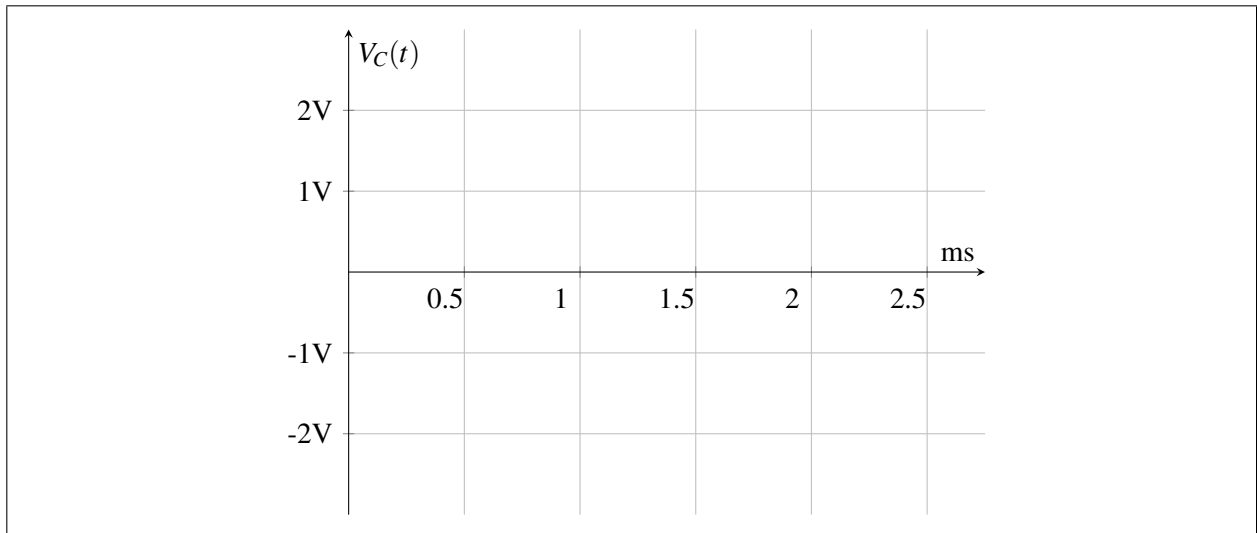


Figure 5.3

Assuming $V_C(0) = 0\text{V}$, plot $V_C(t)$ from $t = 0\text{ms}$ to $t = 2.5\text{ms}$ in the space provided below. Clearly label the minimum and maximum values.



PRINT your student ID: _____

- (c) (4 points) Youbin finds that the difference in peak voltages of V_C is small when the capacitance changes. He decides to amplify V_C by a gain of 5 in order to better distinguish the peak voltages. He designs the circuit shown in Figure 5.4. You may assume the op-amp is ideal. **Choose values for resistors R_1 and R_2 such that $V_{\text{out}} = 5V_C$.** Show your work.

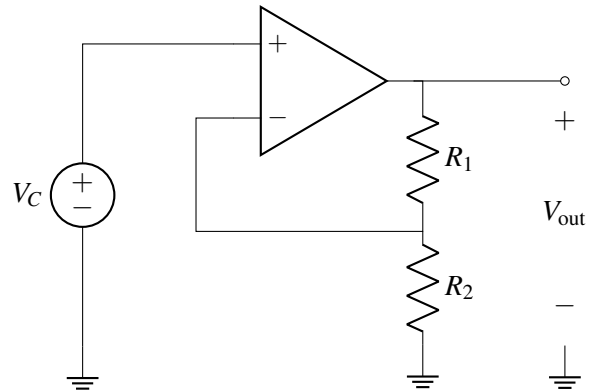


Figure 5.4: Amplifier circuit

PRINT your student ID: _____

6. Let's Go, Mooncow! (28 points)

UC Berkeley, in preparation for their new Space Exploration Research Center at NASA Ames, has tasked you with understanding the space travels of a newly discovered creature named "Mooncow".

(Despite his name, Mooncow bears a surprisingly strong resemblance to what we call "Monkeys" on Earth).



A scientist's depiction
of "Mooncow"

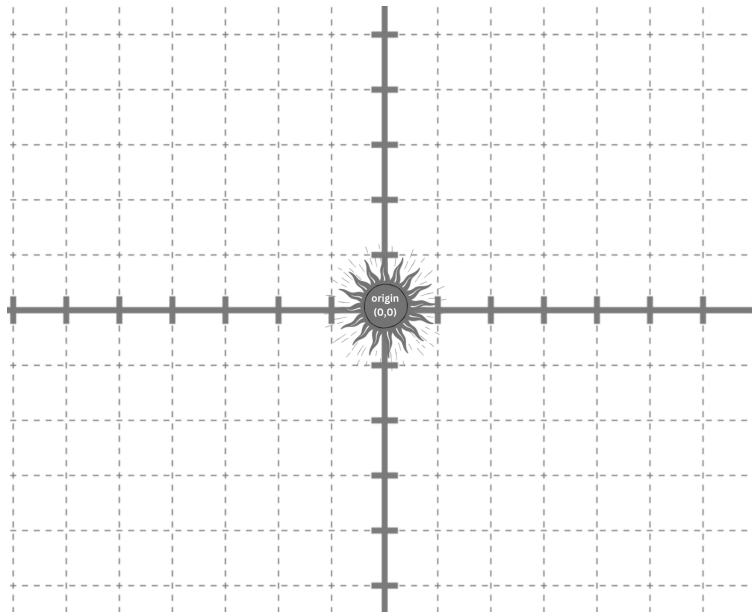
For this problem, assume the galaxy is two-dimensional, and the sun represents the origin.

- (a) (2 points) Mooncow is moving in a 2D galaxy and has access to three boosters. Each booster moves him in a specific direction: $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ 6 \end{bmatrix}$. **He must choose the fewest number of boosters to reach any point in the galaxy. Which boosters should he choose?**

Note: There may be multiple correct answers.

- $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$
- $\begin{bmatrix} -1 \\ -2 \end{bmatrix}$
- $\begin{bmatrix} 5 \\ 6 \end{bmatrix}$

- (b) (3 points) Mooncow wants to plot the locations of two planets. Using the provided graph, **plot the position vectors of the planets he sees: Planet X: $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$, Planet Y: $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$. Label the planets.**



PRINT your student ID: _____

- (c) (4 points) Mooncow sees a solar eclipse taking place on Planet B due to the position of Planet A. He is at $\begin{bmatrix} 4 \\ 0 \end{bmatrix}$, Planet A is at $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and Planet B is at $\begin{bmatrix} 4 \\ -4 \end{bmatrix}$. Mooncow wants to travel to the eclipsed region (i.e. the line segment joining the two planets) as shown in Figure 6.1. Mooncow takes the shortest path to reach this line segment.

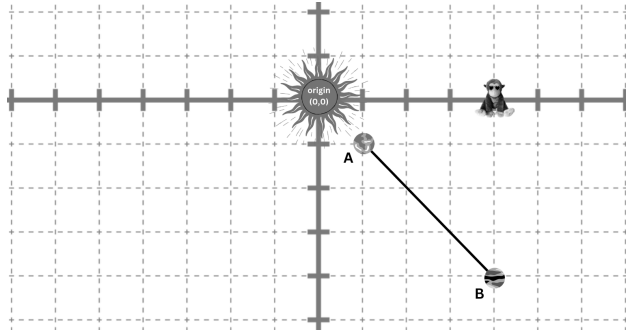


Figure 6.1

Compute the coordinates of where Mooncow will arrive on the path of the eclipse, and state how far he will be from Planet A when he arrives. Your solution must be justified by calculations, but you may use the graph to help you.

PRINT your student ID: _____

- (d) (3 points) Mooncow's position vector is at $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$. He is orbiting the sun in a counterclockwise direction. His velocity vector is in the direction of his motion and is orthogonal to his position vector. **Calculate Mooncow's velocity vector. His velocity vector should be unit length.** Show your work.

PRINT your student ID: _____

- (e) (4 points) Kanav finds Mooncow is located at $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$. He recalibrates his measurement device and finds these measurements need to be rotated **clockwise** by 60 degrees. **Find Mooncow's real location.** Show your work. Recall that $\sin(60^\circ) = \frac{\sqrt{3}}{2}$, $\sin(-60^\circ) = -\frac{\sqrt{3}}{2}$, and $\cos(60^\circ) = \cos(-60^\circ) = \frac{1}{2}$.

PRINT your student ID: _____

- (f) (4 points) We have lost track of Mooncow and are searching the galaxy for him! Anish is located on Planet X at $\begin{bmatrix} 0 \\ 4 \end{bmatrix}$, and he detects Mooncow is 4 units away. Sabriya is located on Planet Y at $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$, and she detects Mooncow is 5 units away. They know Mooncow always stays at least 2 units away from the sun. **What coordinates is Mooncow at?** Show your work. Your solution must be justified by calculations, but you may use the graph to help you.

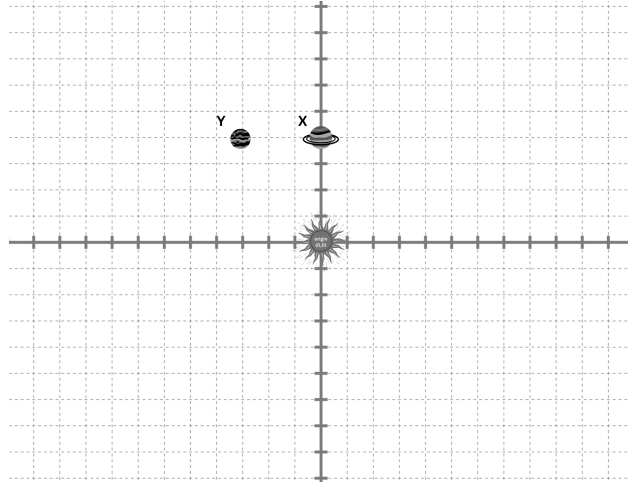


Figure 6.2

PRINT your student ID: _____

- (g) (4 points) Sayan has been tracking Mooncow and has the following measurements for Mooncow's positions:

Table 6.1

| x | y |
|-----|-----|
| -3 | -8 |
| 0 | 10 |
| 5 | 0 |
| 0 | -10 |
| -5 | 0 |
| 4 | 6 |

Kepler's laws dictate that Mooncow's spaceship follows an elliptical orbit. Recall an ellipse follows the formula $\alpha x^2 + \beta xy + \gamma y^2 + \delta x + \epsilon y = 1$. **What are the unknowns Sayan must identify to find the equation for the ellipse? Using the data points in Table 6.1, formulate the least squares equation in matrix-vector form that would be used to solve for the equation of the ellipse.**

PRINT your student ID: _____

- (h) (4 points) To bring Mooncow back home, Anish needs to know Mooncow's mass. He cannot measure Mooncow's mass directly; instead, he measures the gravitational force on Mooncow F and Mooncow's acceleration a and uses the equation $F = ma$ to solve for mass. The measurements of Mooncow's acceleration and force are as follows:

Table 6.2

| $a(\frac{m}{s^2})$ | $F(\frac{kg \cdot m}{s^2})$ |
|--------------------|-----------------------------|
| -2 | -20 |
| -1 | -15 |
| 0 | -3 |
| 1 | 10 |
| 2 | 20 |

We use the equation $F \approx ma$ to relate these variables. **Set up a least squares problem to estimate m . Compute the least squares solution of m .** Show your work.

PRINT your student ID: _____

7. Caterwauls! (18 points)

- (a) (4 points) Thomas' cat Luna frequently wanders off. In order to keep track of her, Thomas is building a tracking system. He installs a tracking collar that transmits a distinct signal \vec{l} shown in Figure 7.1.

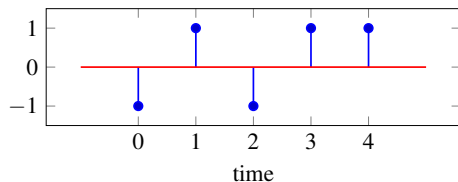


Figure 7.1: Luna's signal \vec{l}

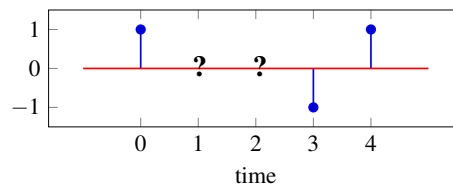


Figure 7.2: Recorded signal \vec{r}

In order to test his system he records the signal \vec{r} , as well as the cross-correlation $\text{corr}_{\vec{r}}(\vec{l})$. Unfortunately, he realizes that \vec{r} has been corrupted in some places, as shown in Figure 7.2. The cross-correlation $\text{corr}_{\vec{r}}(\vec{l})$ is given in Figure 7.3.

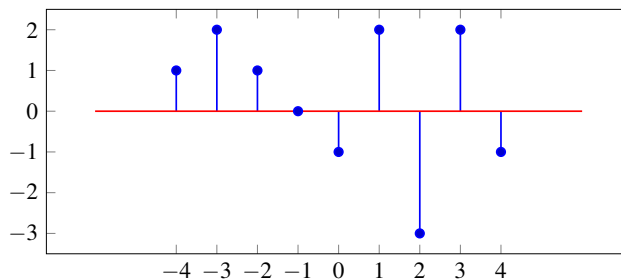


Figure 7.3: Cross-correlation $\text{corr}_{\vec{r}}(\vec{l})$

Recover the missing entries $\vec{r}[1]$ and $\vec{r}[2]$. Show your work.

PRINT your student ID: _____

- (b) (3 points) Luna has wandered off! To locate her, Thomas records the signal \vec{s} transmitted by Luna's collar and computes $\text{corr}_{\vec{s}}(\vec{l})$ shown in Figure 7.4.

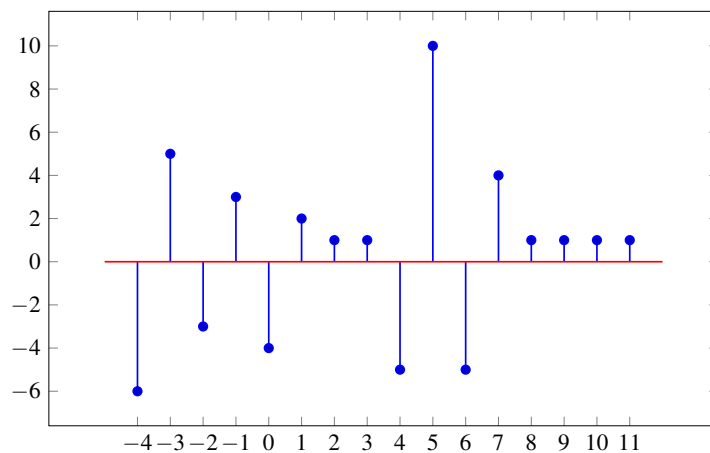


Figure 7.4: Cross-correlation $\text{corr}_{\vec{s}}(\vec{l})$

Assume that the x-axis ticks correspond to a shift of 1×10^{-6} s and the transmissions travel at 3×10^8 m/s. **Compute the distance between Thomas and Luna.** Show your work.

PRINT your student ID: _____

- (c) (4 points) To prevent Luna from wandering too far, Thomas wants to design a circuit that plays a recall sound through a speaker. The volume of the speaker should increase proportionally to Luna's distance from him. Thomas already has a converter circuit that converts Luna's distance to a voltage V_{dist} . The converter circuit and speaker can be represented by the Thevenin equivalents shown in Figures 7.5 and 7.6 respectively.

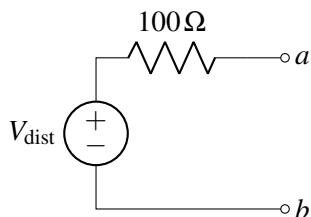


Figure 7.5: Thevenin equivalent of converter

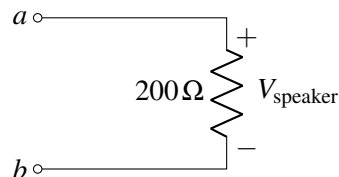


Figure 7.6: Thevenin equivalent of speaker

Thomas first connects the two circuits together directly, shown in Figure 7.7.

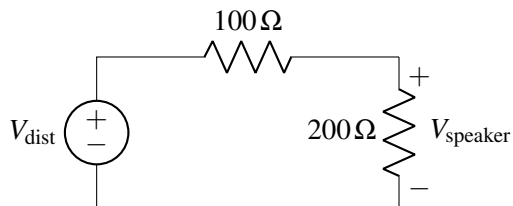
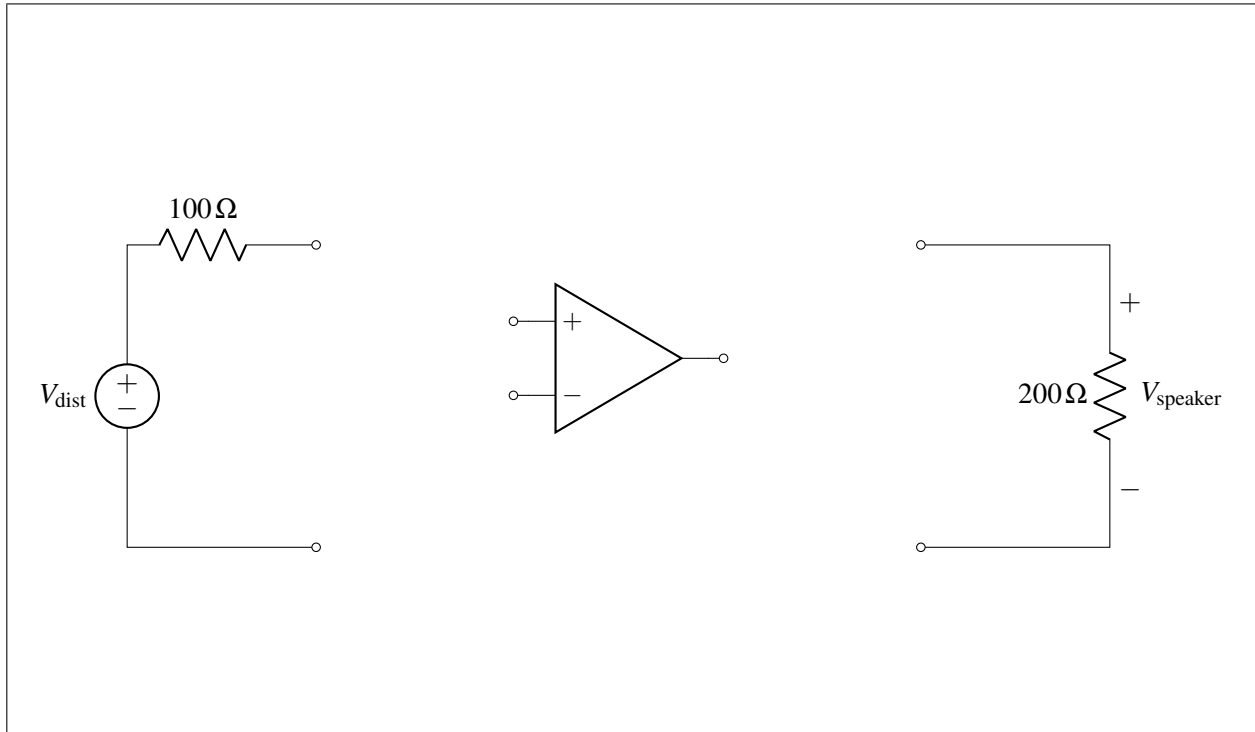


Figure 7.7: Direct connection

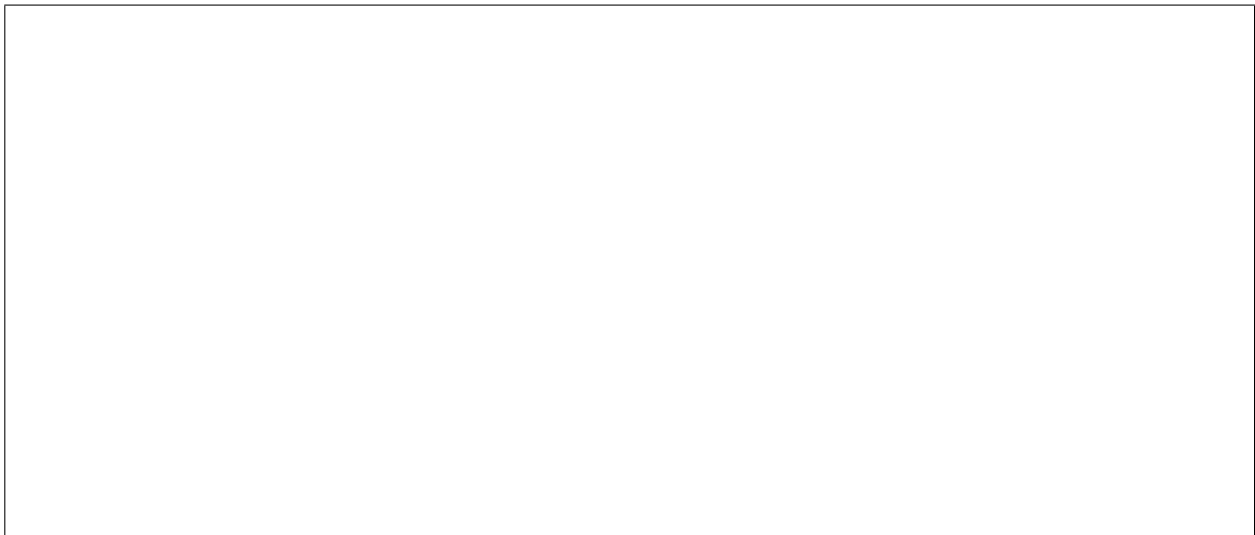
Given that $0\text{ V} \leq V_{\text{dist}} \leq 6\text{ V}$, **what is the maximum power dissipated by the speaker?** Show your work.

PRINT your student ID: _____

- (d) (4 points) Thomas realizes that the speaker volume is too low when directly connected to the converter. He instead wants to connect the circuits such that $V_{\text{speaker}} = V_{\text{dist}}$. He only has access to a single ideal op-amp and no other components. **Complete the circuit below by connecting the elements given. No element terminal should be left unconnected.**



- (e) (3 points) Thomas needs to build a resistor out of resistive cubes which have a length, width, and height of $5 \times 10^{-3}\ \text{m}$ and a resistivity of $8 \times 10^{-3}\ \Omega\text{m}$. He plans to attach the cubes in a line into one long resistor. **How many cubes does he need to make a $40\ \Omega$ resistor?** Justify your answer.

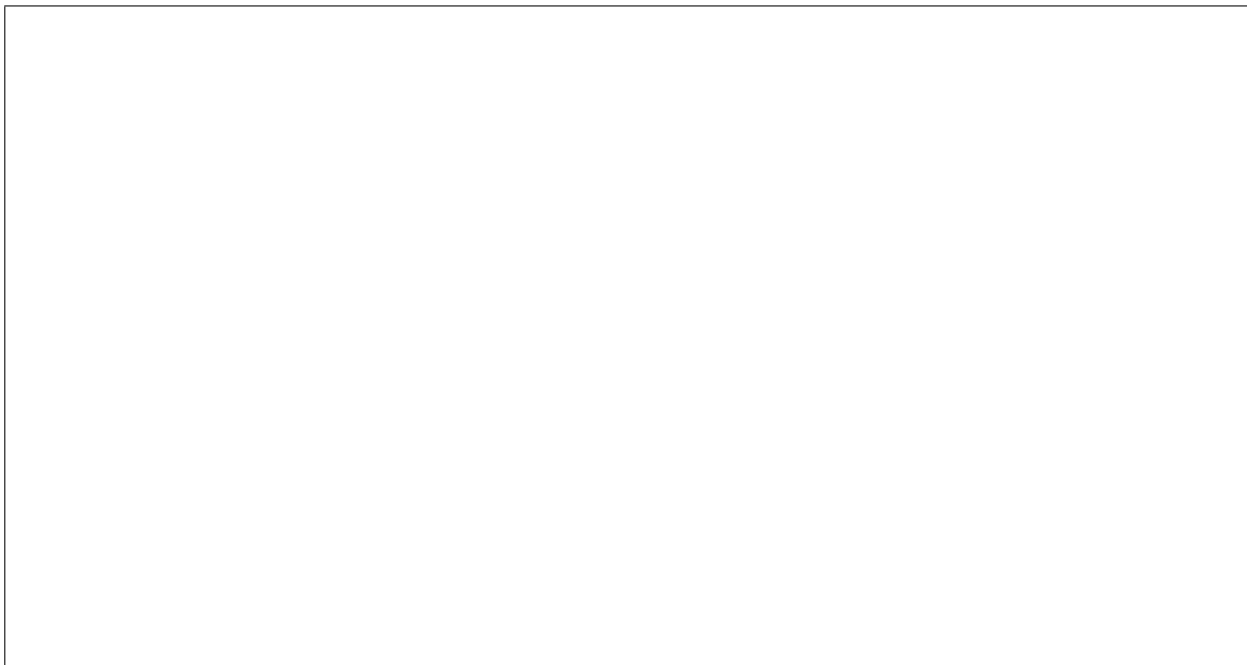


PRINT your student ID: _____

8. Aficiona-dough (25 points)

Jiarui owns two pizza shops: Slice and Cheddarboard. He models the movement of his customers each week. Each timestep represents a week.

- (a) (4 points) Each week 40% of Slice's customers move to Cheddarboard to buy pizza, while the remaining customers stay at Slice. 25% of Cheddarboard customers move to Slice, while the remainder stay at Cheddarboard. **Draw a state transition diagram modeling the flow of customers between Jiarui's restaurants.**



PRINT your student ID: _____

- (b) (4 points) Jiarui observes that the system follows a new state transition diagram (due to a change in his menu), which is given in Figure 8.1.

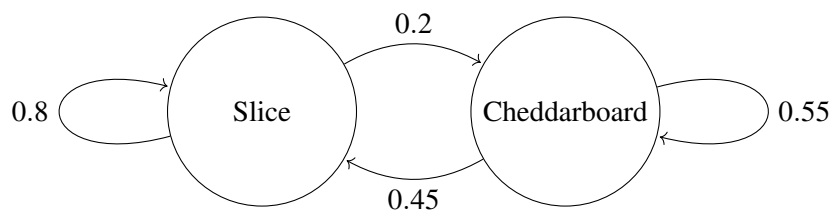


Figure 8.1: New state transition diagram of the system.

Write the state transition matrix \mathbf{P} corresponding to the Figure 8.1, such that $\vec{c}[t+1] = \mathbf{P} \cdot \vec{c}[t]$

where $\vec{c}[t] = \begin{bmatrix} \text{Slice}[t] \\ \text{Cheddarboard}[t] \end{bmatrix}$.

What is the nullspace of \mathbf{P} ? Justify your answer.

Hint: You need not mathematically compute the nullspace.

PRINT your student ID: _____

- (c) (6 points) Jiarui opens a third pizza shop: Asparagus. Initially, he has a total of 120 customers. The state transition matrix of the system describing the flow of customers between the three restaurants is:

$$\mathbf{Q} = \begin{bmatrix} \frac{2}{3} & \frac{2}{5} & \frac{1}{2} \\ 0 & \frac{3}{5} & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} \end{bmatrix}$$

such that, $\vec{c}[t+1] = \mathbf{Q} \cdot \vec{c}[t]$ where $\vec{c}[t] = \begin{bmatrix} \text{Slice}[t] \\ \text{Cheddarboard}[t] \\ \text{Asparagus}[t] \end{bmatrix}$.

Find the number of customers in each shop at steady state. Show your work.

PRINT your student ID: _____

(d) (6 points) Jiarui again observes a change in his system, and finds the new state transition matrix is

$$\mathbf{R} = \begin{bmatrix} \frac{5}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{4}{3} & \frac{4}{3} & \frac{1}{3} \\ \frac{4}{3} & \frac{1}{3} & \frac{4}{3} \end{bmatrix}.$$

Show that the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of this matrix. What is the corresponding eigenvalue?

Assuming that the initial state is $\begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix}$, how many customers are there in each shop after 10 timesteps? You do not need to reduce your answer. Show your work.

PRINT your student ID: _____

- (e) (5 points) Let a state transition matrix \mathbf{S} have eigenvalues $\lambda_1 = 1$, $\lambda_2 = 2$ and $\lambda_3 = \frac{1}{2}$ corresponding to eigenvectors \vec{v}_1 , \vec{v}_2 and \vec{v}_3 respectively. The initial state is given by

$$\vec{c}[0] = \alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \alpha_3 \vec{v}_3,$$

where $\alpha_1, \alpha_2, \alpha_3 \in \mathbb{R}$. Let $\vec{c}[t]$ represent the state after t timesteps.

Write $\vec{c}[t]$ in terms of α_i , λ_i and \vec{v}_i , where $i = 1, 2, 3$.

Under what conditions on α_1 , α_2 and α_3 is $\lim_{t \rightarrow \infty} \vec{c}[t]$ finite? Justify your answer.

PRINT your student ID: _____

9. Proofs (15 points)

- (a) (7 points) Consider matrices $\mathbf{A} \in \mathbb{R}^{n \times n}$ and $\mathbf{B} \in \mathbb{R}^{m \times n}$. Assume that \mathbf{A} is invertible and \mathbf{B} has a non-trivial nullspace. **Prove that \mathbf{BA} has a nontrivial nullspace.**

PRINT your student ID: _____

- (b) (8 points) Let $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_k$ be k matrices in $\mathbb{R}^{n \times n}$. Assume all \mathbf{A}_i have $\vec{v} \in \mathbb{R}^n$ as an eigenvector, with corresponding eigenvalue λ_i for $i = 1, 2, \dots, k$. Assume that $\sum_{i=1}^k \lambda_i \neq 1$, and the matrix $(\mathbf{I} - \sum_{i=1}^k \mathbf{A}_i)$ is invertible, where \mathbf{I} is the identity matrix in $\mathbb{R}^{n \times n}$. **Prove that \vec{v} is an eigenvector of $(\mathbf{I} - \sum_{i=1}^k \mathbf{A}_i)^{-1}$. What is the corresponding eigenvalue?** Show your work.

PRINT your student ID: _____

Extra page for scratchwork.

If you want any work on this page to be graded, explicitly tell us to grade this page number in the original problem's solution area.

PRINT your student ID: _____

10. Orthonormal Least Squares (13 points)

(a) (5 points) Suppose we are given the matrix

$$\mathbf{A} = \begin{bmatrix} | & | \\ \vec{a}_1 & \vec{a}_2 \\ | & | \end{bmatrix},$$

where $\|\vec{a}_1\| = \|\vec{a}_2\| = 1$ and \vec{a}_1 is orthogonal to \vec{a}_2 , i.e., $\vec{a}_1 \perp \vec{a}_2$. **Show that \vec{a}_1 and \vec{a}_2 are linearly independent.**

Hint: Consider a proof by contradiction (assume \vec{a}_1 and \vec{a}_2 are linearly dependent, i.e., $\vec{a}_1 = \beta \vec{a}_2$ for $\beta \in \mathbb{R}$).

PRINT your student ID: _____

(b) (8 points) Now suppose that the matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$ is such that

$$\begin{bmatrix} | & | & \cdots & | \\ \vec{a}_1 & \vec{a}_2 & \cdots & \vec{a}_n \\ | & | & & | \end{bmatrix},$$

where $\|\vec{a}_1\| = \|\vec{a}_2\| = \cdots = \|\vec{a}_n\| = 1$ and $\vec{a}_1, \dots, \vec{a}_n$ are pairwise mutually orthogonal, i.e. $\vec{a}_i \perp \vec{a}_j$ for all $i, j = 1, \dots, n$ and $i \neq j$. For $\vec{b} \in \mathbb{R}^m$, we are given $\langle \vec{a}_i, \vec{b} \rangle = c_i$ for $i = 1, \dots, n$. **Find the projection of \vec{b} onto $\text{Col}(\mathbf{A})$** , where $\text{Col}(\mathbf{A})$ represents the column space of \mathbf{A} . **Write your answer in terms of \vec{a}_i and c_i .** Show your work.

Hint: The projection of \vec{b} onto $\text{Col}(\mathbf{A})$ is given by $\mathbf{A}\vec{x}$ where \vec{x} is the least squares solution of $\mathbf{A}\vec{x} = \vec{b}$.

PRINT your student ID: _____

Extra page for scratchwork.

If you want any work on this page to be graded, explicitly tell us to grade this page number in the original problem's solution area.

PRINT your student ID: _____

11. Gold Code Inner Product Circuits (27 points)

Recall that Gold codes are sequences with elements equal to ± 1 . We often need to compare the similarity of Gold codes by finding their inner product. In this problem, we will try to design a circuit that can compute inner products of Gold codes.

- (a) (2 points) We are given two Gold codes $\vec{s}_1 = [1 \ -1 \ -1 \ -1 \ 1 \ 1]^T$ and $\vec{s}_2 = [1 \ 1 \ -1 \ -1 \ 1 \ -1]^T$ each of length 6. The codes are represented by time-varying voltage signals $V_1(t), V_2(t)$ that map the ± 1 elements to ± 1 V symbols of length 1 ms as shown in Figure 11.1.

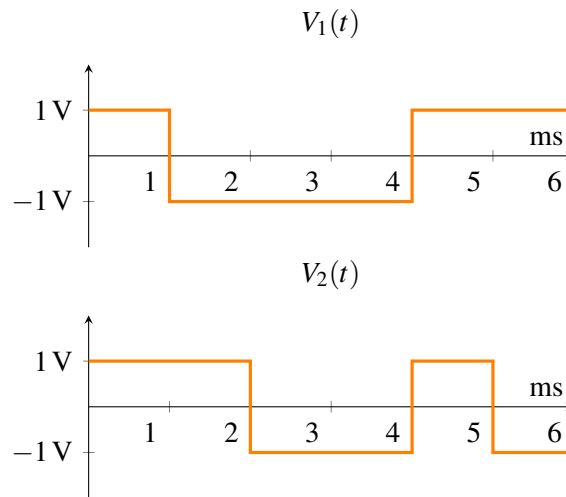


Figure 11.1: Time-varying voltage signals $V_1(t), V_2(t)$ that represent \vec{s}_1, \vec{s}_2 respectively

Compute the inner product $\langle \vec{s}_1, \vec{s}_2 \rangle$. Show your work.

PRINT your student ID: _____

- (b) (8 points) For Gold code signals, we notice that the multiplication of ± 1 elements is equivalent to checking if the two elements are equal. In the inner product circuit shown in Figure 11.2, an inverting summer and a match detect circuit are used to check when $V_1(t) = V_2(t)$. The results from the match detect circuit are then integrated across the length of the signal to produce V_{IP} which represents the final inner product value.

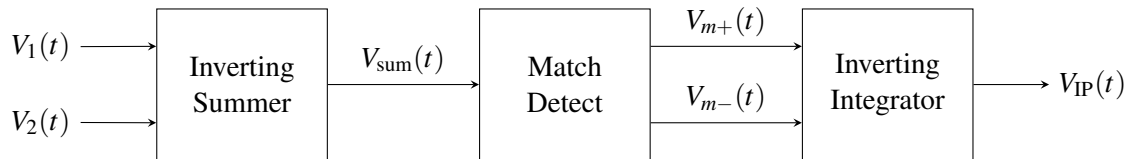
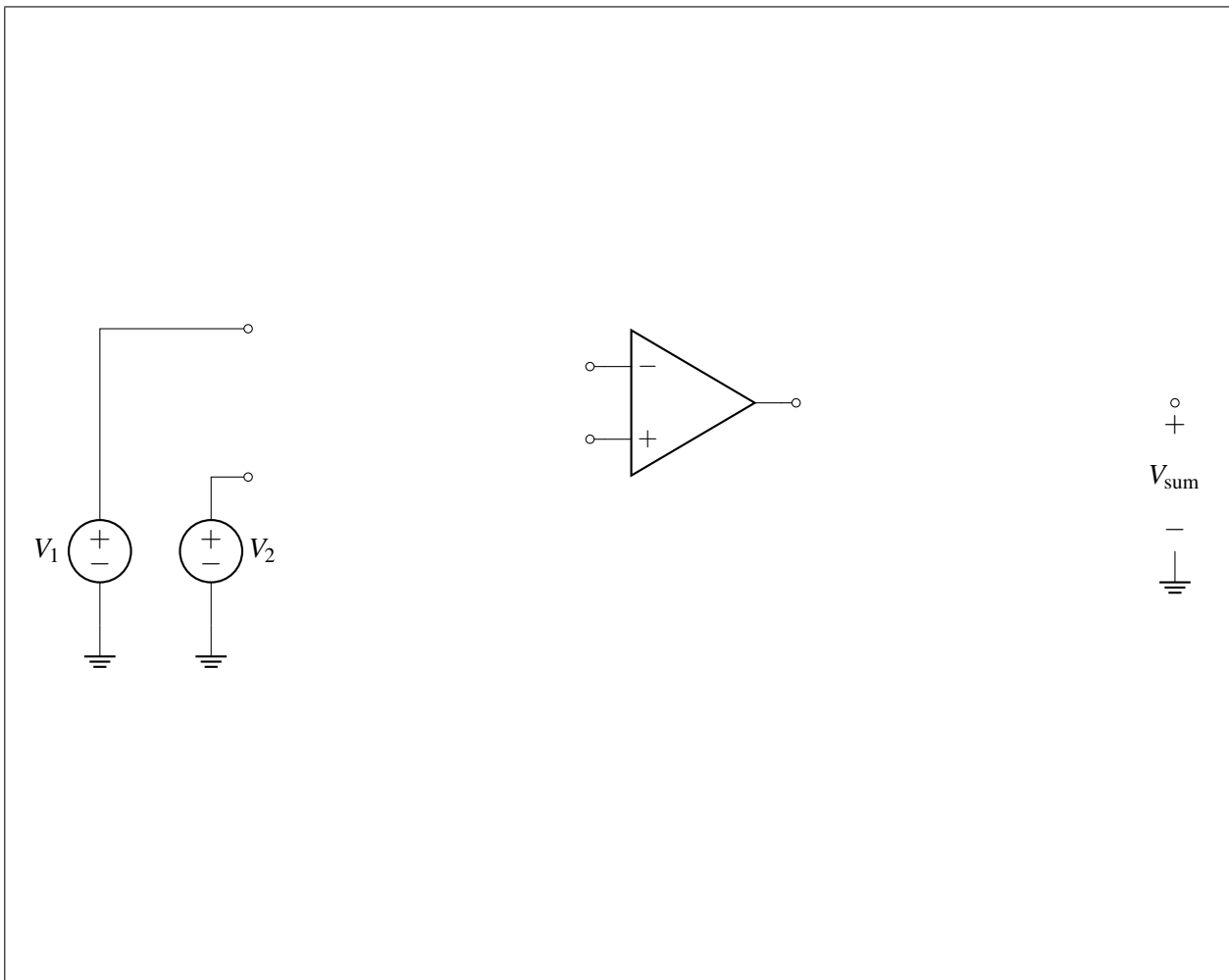


Figure 11.2: Block diagram of inner product circuit

In this part, we wish to design the inverting summer block. We have access to a single ideal op-amp (already drawn) and up to three resistors for which we can choose values. No other components are available. **Design a circuit such that $V_{sum} = -V_1 - V_2$. Label the resistances for all resistors used.**



PRINT your student ID: _____

- (c) (5 points) The match detect circuit wants to use $V_{\text{sum}} = -V_1 - V_2$ to determine when $V_1 = V_2$. When $V_{\text{sum}} = 2\text{ V}$, we know that V_1 and V_2 match with value -1 V . When $V_{\text{sum}} = -2\text{ V}$, we know that V_1 and V_2 match with value 1 V . The match detect circuit can be implemented using comparators with outputs V_{m+} and V_{m-} as shown in Figure 11.4.

| V_{sum} | V_{m+} | V_{m-} |
|------------------|----------|----------|
| 2 V | 1 V | -3 V |
| 0 V | 1 V | 1 V |
| -2 V | -3 V | 1 V |

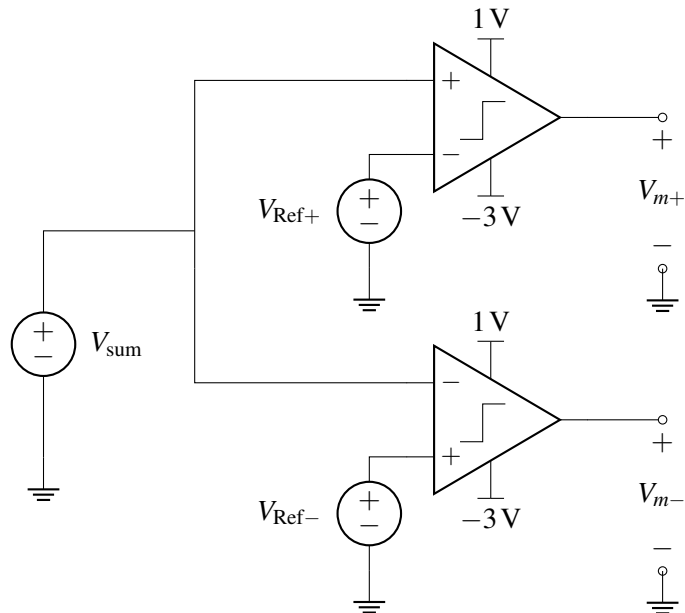


Figure 11.3: Input output table

Figure 11.4: Match detect circuit

Choose values for $V_{\text{Ref}+}$ and $V_{\text{Ref}-}$ such that $V_{\text{sum}}, V_{m+}, V_{m-}$ satisfy the table in Figure 11.3. Justify your answer.

PRINT your student ID: _____

- (d) (7 points) In order to compute the inner product, we can use the circuit in Figure 11.5 to integrate the match signals. You may assume the op-amp is ideal.

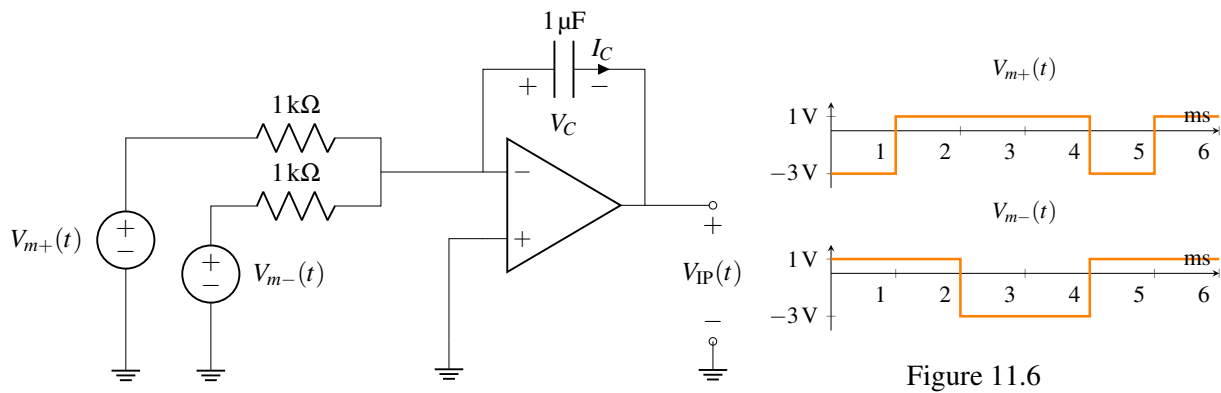
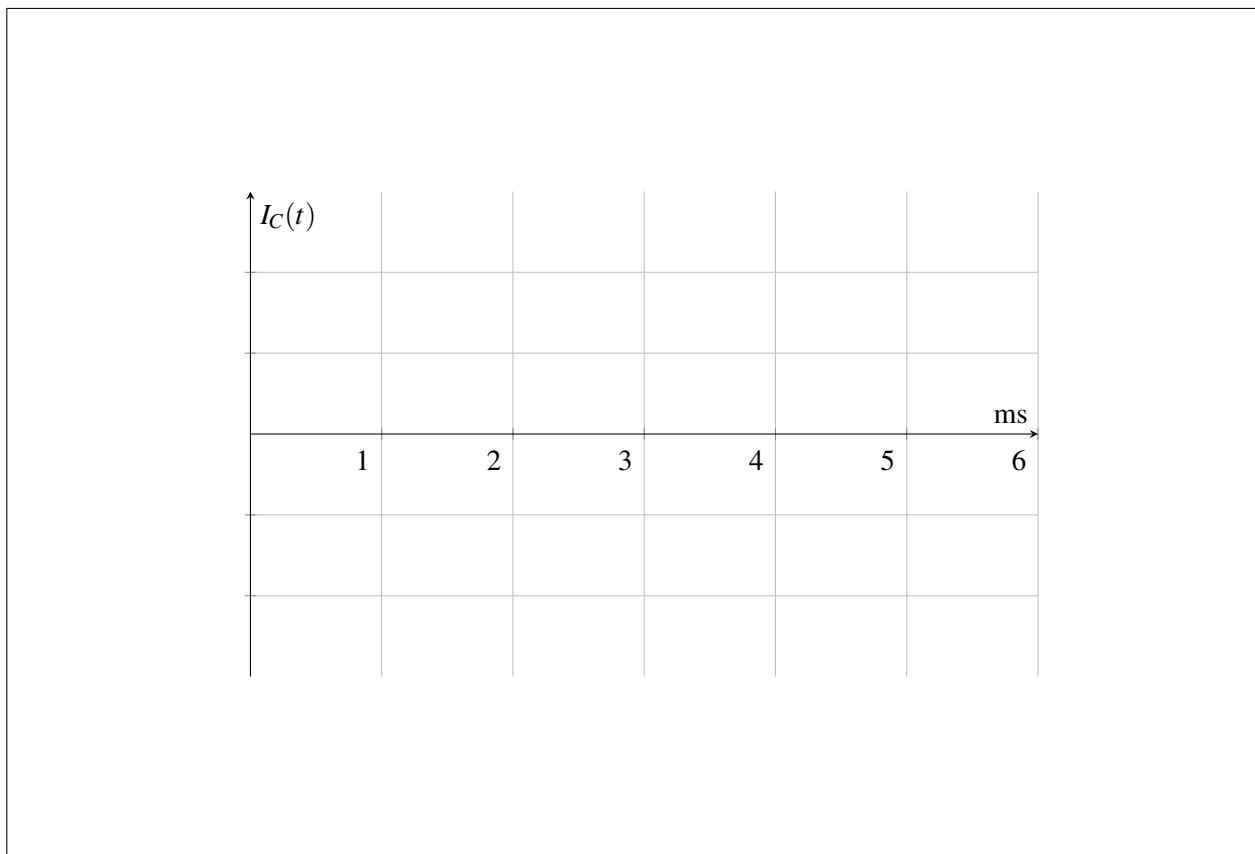


Figure 11.5: Inverting integrator circuit

The waveforms for $V_{m+}(t)$ and $V_{m-}(t)$ are given in Figure 11.6.

- Plot $I_C(t)$ from $t = 0$ ms to $t = 6$ ms in the graph provided. Label the units and current values in your graph.
- Compute $V_{IP}(6\text{ms})$. Assume that $V_{IP}(0) = 0\text{V}$.

Show your work.



PRINT your student ID: _____

- (e) (5 points) As we increase the signal length, the maximum/minimum value of V_{IP} also increases. In order to keep the output voltage to a manageable level, we decide to switch out the $1\ \mu\text{F}$ capacitor for a variable capacitor shown in Figure 11.7.

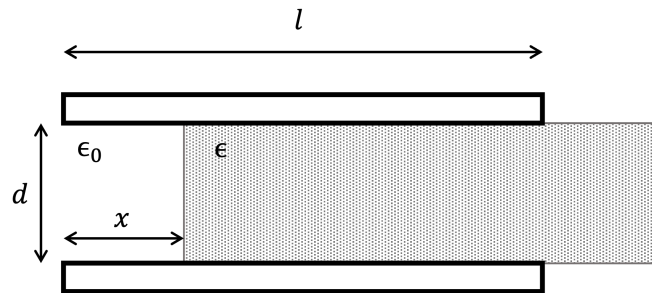


Figure 11.7: Variable capacitor

The capacitor has square plates with **length and width l with a separation of d** . Inside, we have a dielectric material with **permittivity $\epsilon = 5\epsilon_0$** that we can slide to change the total capacitance between the plates. x measures the displacement of the dielectric material. Assuming $0 \leq x \leq l$, **find the total capacitance C in terms of l, d, x, ϵ_0** . You do not need to reduce your answer. Show your work.

PRINT your student ID: _____

Extra page for scratchwork.

If you want any work on this page to be graded, explicitly tell us to grade this page number in the original problem's solution area.

PRINT your student ID: _____

Extra page for scratchwork.

If you want any work on this page to be graded, explicitly tell us to grade this page number in the original problem's solution area.

PRINT your student ID: _____

Extra page for scratchwork.

If you want any work on this page to be graded, explicitly tell us to grade this page number in the original problem's solution area.

PRINT your student ID: _____

Extra page for scratchwork.

If you want any work on this page to be graded, explicitly tell us to grade this page number in the original problem's solution area.

PRINT your student ID: _____

Doodle page!

Draw us something if you want or give us suggestions, compliments, or complaints.
You can also use this page to report anything suspicious that you might have noticed.

Read the following instructions before the exam.

There are 11 problems of varying numbers of points. Not all subparts of a question are related to each other. You have 170 minutes for the exam. The problems are of varying difficulty, so pace yourself accordingly and avoid spending too much time on any one question until you have gotten all of the other points you can.

There are 40 pages on the exam, so there should be 20 sheets of paper in the exam. The exam is printed double-sided. Do not forget the problems on the back sides of the pages! Notify a proctor immediately if a page is missing. **Do not tear out or remove any of the pages. Do not remove the exam from the exam room.**

No collaboration is allowed, and do not attempt to cheat in any way. Cheating will not be tolerated.

Write your student ID on each page before time is called. If a page is found without a student ID, we are not responsible for identifying the student who wrote that page.

You may consult THREE handwritten $8.5'' \times 11''$ note sheet (front and back). No phones, calculators, tablets, computers, other electronic devices, or scratch paper are allowed.

Please write your answers legibly in the boxed spaces provided on the exam. The space provided should be adequate. **If you still run out of space, please use a blank page and clearly tell us in the original problem space where to look for your solution.**

Show all of your work in order to receive full credit. Partial credit will be given for substantial progress on each problem.

If you need to use the restrooms during the exam, bring your student ID card, your phone, and your exam to a proctor. You can collect them once you return from the restrooms.

Our advice to you: if you can't solve the problem, state and solve a simpler one that captures at least some of its essence. You might get some partial credit, and more importantly, you will perhaps find yourself on a path to the solution.

Good luck!

Do not turn this page until the proctor tells you to do so.