EECS 16A Designing Information Devices and Systems I
Spring 2023 Final (170 minutes)

PRINT your student ID: ________________________________

PRINT AND SIGN your name: ____________________, ____________________ ________________________
(last name) (first name) (signature)

PRINT the time of your discussion section and your GSI's name: ________________________________

PRINT the student IDs of the person sitting on your right: ____________________ and left: ____________________

General Notes

- This exam has a combination of multiple choice questions and fill in the blank.
- This exam will be partially autograded. **You must adhere to the following format to receive full credit:**
  - For fill in the blank questions, **legibly write your final answer entirely in the provided boxes**.
  - For questions with **circular bubbles**, select exactly *one* choice, by filling the bubble ●.
    ○ You must choose either this option. ○ Or this one, but not both!
  - For questions with **square boxes**, you may select *multiple* choices, by filling the squares ■.
    ☐ You could select this choice. ☐ You could select this one too!

1. HONOR CODE

Please read the following statements of the honor code, and sign your name (you don't need to copy it).

I will respect my classmates and the integrity of this exam by following this honor code. I affirm:

- *I have read the instructions for this exam. I understand them and will follow them.*
- *All of the work submitted here is my original work.*
- *I did not reference any sources other than my unlimited printed resources.*
- *I did not collaborate with any other human being on this exam.*

Tell us about something you are looking forward to this summer! (1 point) *All answers will be awarded full credit.*

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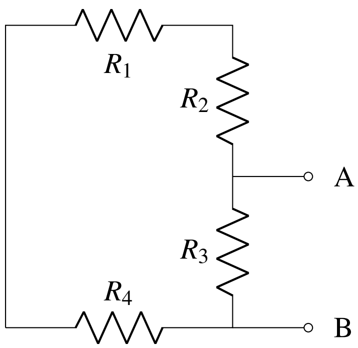
2. Circuit Networks (3 points)

For each part, find the equivalent resistance or capacitance between terminals A and B.

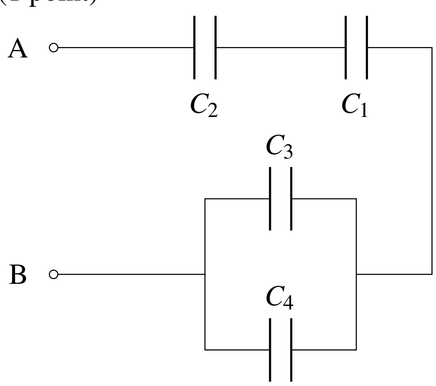
You may use the || operator in your final expressions.

(a) (1 point)



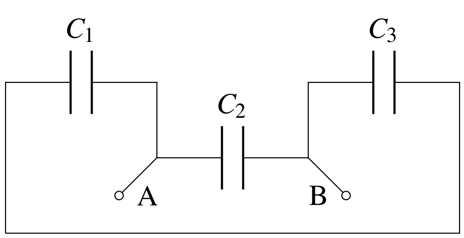
$R_{eq} =$

(b) (1 point)



$C_{eq} =$

(c) (1 point)



$C_{eq} =$

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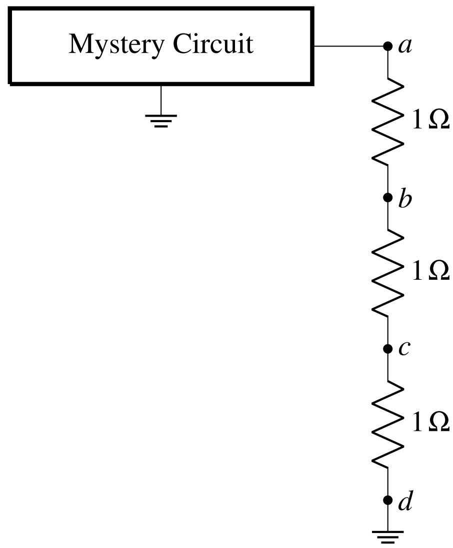
3. Mystery Circuit Voltage Divider (6 points)

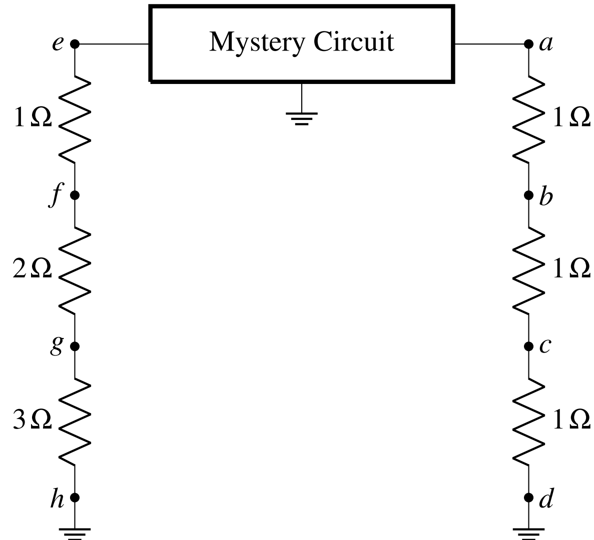
You are given a "mystery circuit" in a box with a part sticking out, as in the diagram below.



(a) (2 points) The voltage at node a is measured to be 6 V. Find the voltages at nodes b and c.

Node b voltage: [ ] V   Node c voltage: [ ] V

(b) (2 points) Imagine there's another part of the circuit added on the left, with differently valued resistors as shown.



Given that both node a and node e are measured to be 6 V, compare the node voltages below.
Use either: $>$, $=$, or $<$.

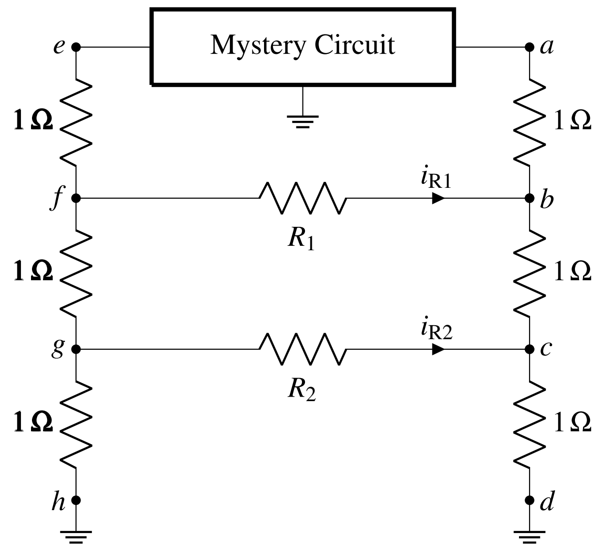
i. e [=] a

ii. f [ ] b

iii. g [ ] c

iv. h [ ] d

(c) (2 points) **Now the resistors on the left side are all adjusted to be equal to $1\,\Omega$**, and an additional resistor $R_1 = 0.5\,\Omega$ is connected between nodes b and f, and resistor $R_2 = 2\,\Omega$ is connected between nodes c and g. You again measure the voltage at both node a and node e to be $6\,\text{V}$.



i. What is the current i_{R1} through resistor R_1?

$i_{R1} =$ ☐ A

ii. Select the correct statement regarding the relative values of i_{R1} and i_{R2}.

○ $i_{R1} > i_{R2}$ ○ $i_{R1} = i_{R2}$ ○ $i_{R1} < i_{R2}$

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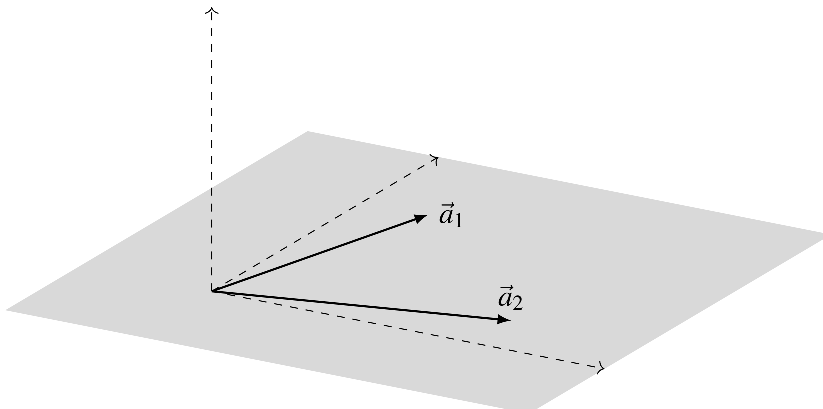
4. Least Squares in Pictures (9 points)

(a) (1 point) Consider a 3×2 matrix $\mathbf{A}$ with linearly independent columns $\vec{a}_1, \vec{a}_2 \in \mathbb{R}^3$. Below we plot $\vec{a}_1$ and $\vec{a}_2$ as well as the plane that they both lie in (the dashed lines are the coordinate axes).



Select all of the following options which describe the plane that contains $\vec{a}_1$ and $\vec{a}_2$ (the shaded region):

☐ Col($\mathbf{A}$), the columnspace of A.

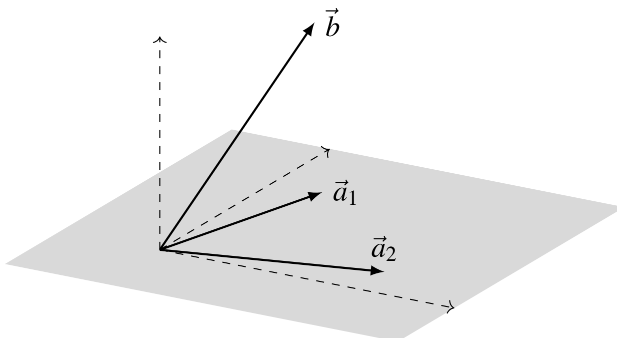
☐ Null($\mathbf{A}$), the nullspace of A.

☐ Det($\mathbf{A}$), the determinant of A.

☐ $\{\mathbf{A}\vec{x} \mid \vec{x} \in \mathbb{R}^2\}$.

(b) (2 points) Now let's take our setup from part (a) and add an additional vector $\vec{b} \in \mathbb{R}^3$ to the diagram. For each of the following choices of $\vec{b}$, select whether the equation $\mathbf{A}\vec{x} = \vec{b}$ has an *exact* solution $\vec{x}$.

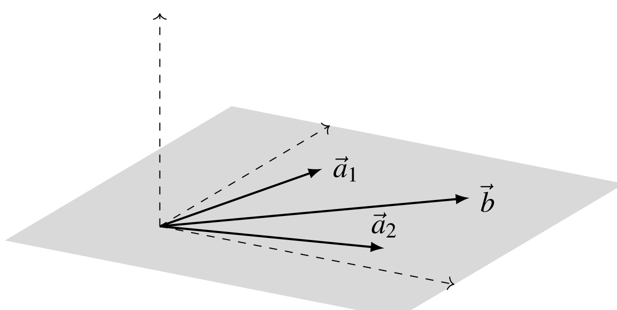
(i). $\vec{b}$ is *not* in the plane



◯ Yes, there exists an $\vec{x}$ such that $\mathbf{A}\vec{x} = \vec{b}$.

◯ No, there does *not* exist an $\vec{x}$ such that $\mathbf{A}\vec{x} = \vec{b}$.

(ii). $\vec{b}$ is in the plane

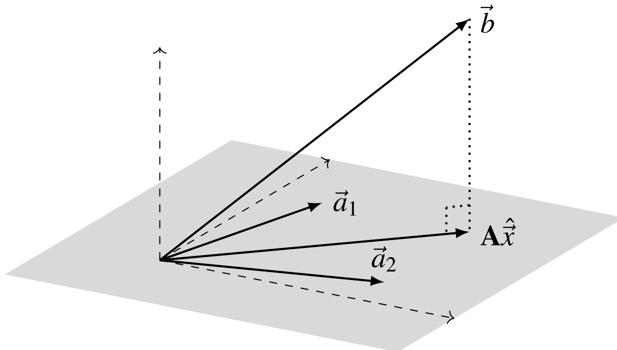


◯ Yes, there exists an $\vec{x}$ such that $\mathbf{A}\vec{x} = \vec{b}$.

◯ No, there does *not* exist an $\vec{x}$ such that $\mathbf{A}\vec{x} = \vec{b}$.

(c) (3 points) Now let's take our setup from the previous parts and add one more vector $\mathbf{A}\hat{\vec{x}}$. Determine in each of the following pictures whether the $\hat{\vec{x}}$ from the given $\mathbf{A}\hat{\vec{x}}$ vector is the least squares solution to $\mathbf{A}\vec{x} \approx \vec{b}$ or not.

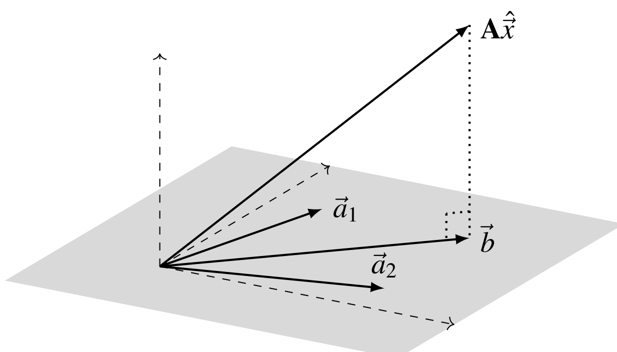
(i). $\vec{b}$ is *not* in the plane. $\mathbf{A}\hat{\vec{x}}$ is in the plane.



◯ Yes, $\hat{\vec{x}}$ is the least squares solution.

◯ No, $\hat{\vec{x}}$ is *not* the least squares solution.

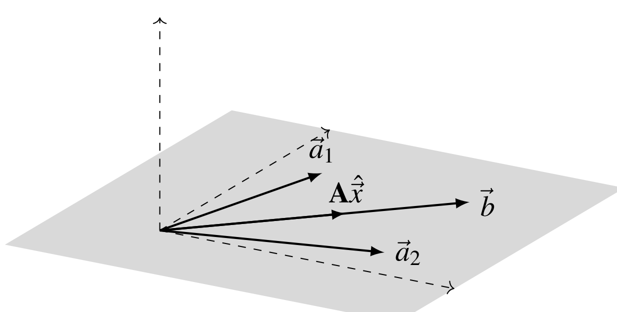
(ii). $\vec{b}$ is in the plane. $\mathbf{A}\hat{\vec{x}}$ is *not* in the plane.



◯ Yes, $\hat{\vec{x}}$ is the least squares solution.

◯ No, $\hat{\vec{x}}$ is *not* the least squares solution.

(iii). $\vec{b}$ is in the plane. $\mathbf{A}\hat{\vec{x}}$ is in the plane and ends at the tip of its arrow (not all the way to the tip of $\vec{b}$).



◯ Yes, $\hat{\vec{x}}$ is the least squares solution.

◯ No, $\hat{\vec{x}}$ is *not* the least squares solution.

(d) (3 points) Suppose that you have found the least squares solution $\hat{\vec{x}}$ to the equation $\mathbf{A}\vec{x} \approx \vec{b}$. Which of the following statements are always true for any $\mathbf{A}$ and $\vec{b}$?

☐ $\vec{b}$ is orthogonal to $\hat{\vec{x}}$

☐ $\vec{b}$ is orthogonal to $A\hat{\vec{x}}$

☐ $\vec{b} - \mathbf{A}\hat{\vec{x}}$ is orthogonal to $\vec{a}_1$

☐ $\vec{b} - \mathbf{A}\hat{\vec{x}}$ is orthogonal to every vector $\vec{y} \in \mathrm{Col}(\mathbf{A})$

☐ $\mathbf{A}\hat{\vec{x}}$ is orthogonal to $\vec{a}_2$

☐ $\mathbf{A}\hat{\vec{x}}$ is parallel or anti-parallel to $\vec{b}$

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5. Battleships (12 points)

You are working on a top-secret project to predict the distance you are from enemy ships in the ocean. You want to build a model that makes these predictions based on a variety of factors, such as water temperature and currents. In particular, you want to design a linear function by choosing the best values of c_1, c_2:

$$d = c_1 m_1 + c_2 m_2$$

where m_1, m_2 correspond to measurements of temperature and currents, and d represents your prediction of the enemy ship's distance.

(a) (3 points) You begin by taking the following measurements:

m_1	m_2	d
0	1	0
1	1	1
2	4	-2

You want to use this data to solve for the best parameters of your model. Find $\mathbf{A}$ and $\vec{b}$ such that your problem is in the form $\mathbf{A}\vec{x} = \vec{b}$, where $\vec{x} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$, the parameters of the model to solve for.

$\mathbf{A} =$

$\vec{b} =$

(b) (5 points) Suppose your colleague took their own measurements, and came up with the following $\mathbf{A}$ matrix and $\vec{b}$ vector.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\vec{b} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Explicitly solve for the least-squares solution $\hat{\vec{x}}$.

$\hat{\vec{x}} =$

(c) (4 points) Your boss attempts to calculate the least squares solution on their own and arrives at

$$\hat{\vec{x}} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Compute the squared error that this model achieves on the data, which is repeated below for your convenience.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hint: The squared error is defined as the squared norm of the error vector.

squared error =

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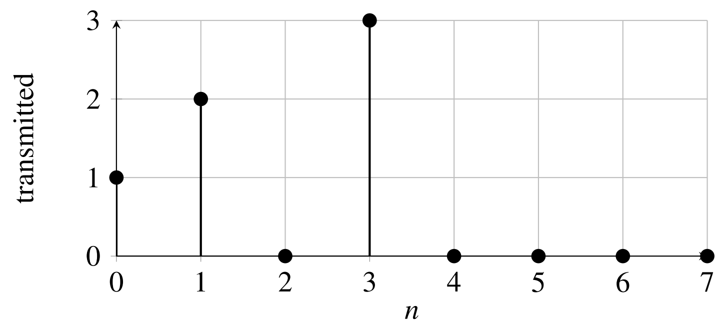
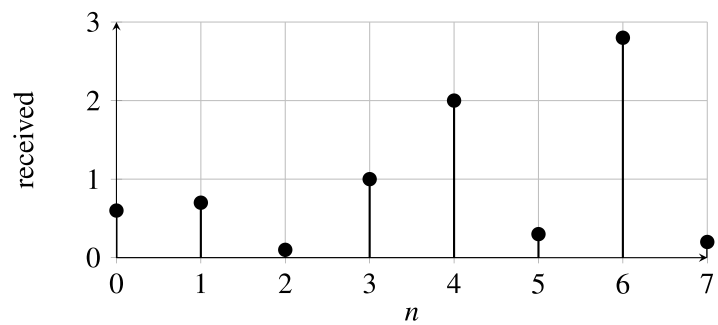
6. 16Aliens (12 points)

Shuming is stranded on an alien planet and can only receive sound signals from distant signal towers. Can you help him find his way back to the launchpad?

(a) (2 points) One of the towers transmits the following signal:



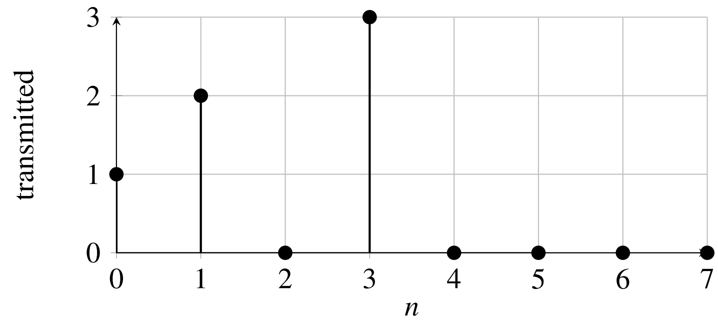
Shuming receives the following noisy signal:

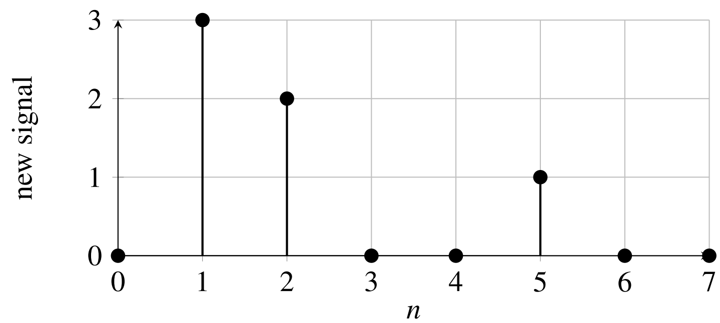


How long did it take for the tower's transmission to reach Shuming? (Assume that the units of n are seconds.)

[ ] seconds

(b) (2 points) Shuming received another signal!





Calculate the cross-correlation between the new signal and the same transmitted signal delayed by 1, i.e. $corr_{new\ signal}(transmitted)[1]$.

$corr_{new\ signal}(transmitted)[1] =$ ☐

Note: $corr_{\vec{x}}(\vec{y})[k] = crosscorr(\vec{x}, \vec{y})[k]$.

(c) (1 point) Shuming receives another signal, and detects a match with the tower's signal with a time delay of $n = 20$ seconds. Given that the speed of sound is 350 meters/second, how far away is the signal tower from Shuming?

meters

(d) (3 points) After performing cross-correlation on the signals, Shuming obtains the following absolute distances between him and each of the towers. Using the provided distances d and tower positions (x, y), write the corresponding **nonlinear** equations for each tower that we could use to solve for Shuming's location. Use r_x and r_y as the variables that we would solve for Shuming's location.

Tower	(x, y)	d
0	(0,0)	1
1	(3,1)	3
2	(-2,1)	2

(e) (4 points) Lastly, linearize your nonlinear equations, and write the final system of linear equations in matrix-vector form. You don't need to solve the system for r_x and r_y.

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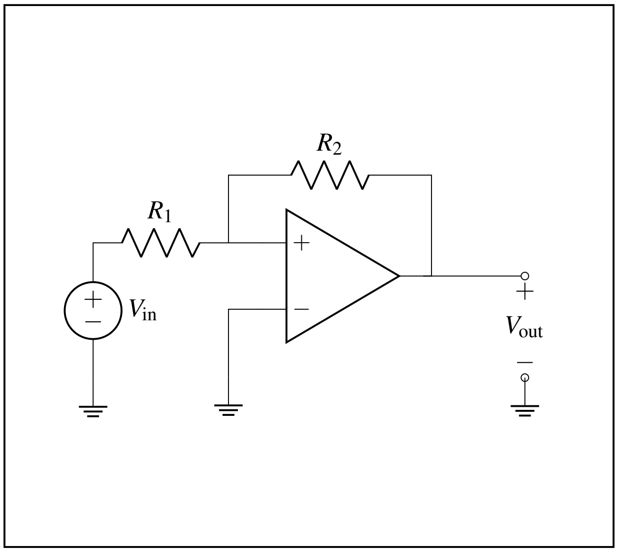
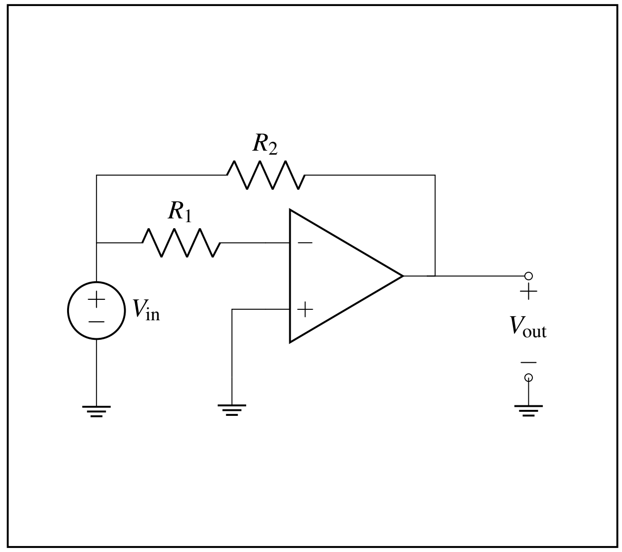
7. Negative Feedback (8 points)

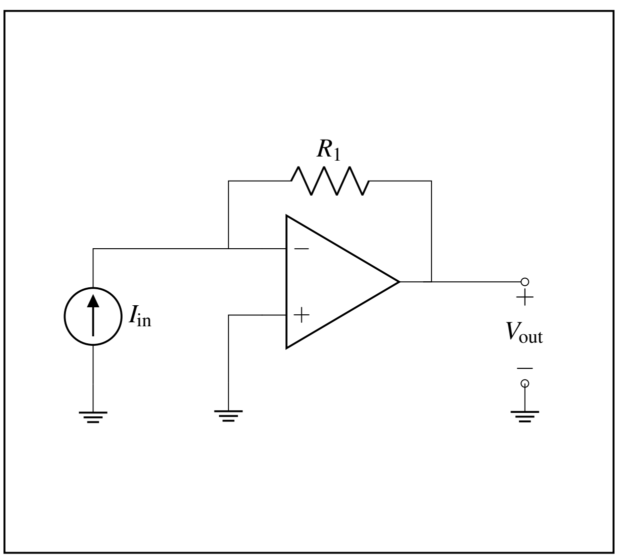
Which of the following op-amp circuits are in negative feedback?



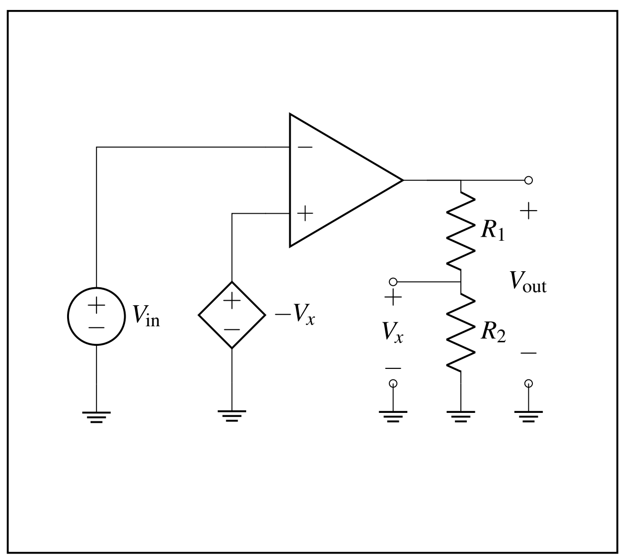
☐



☐



☐



☐

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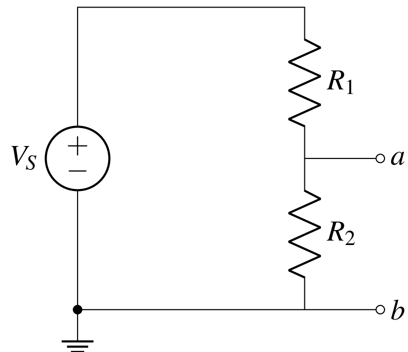
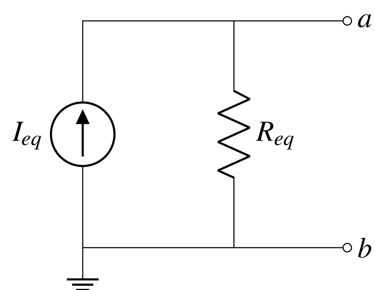
8. What's the Equivalent? (12 points)

(a) (4 points) Solve for the values of I_{eq}, R_{eq} such that the $I-V$ characteristics between terminals a and b are the same in both circuits. Express your answers in terms of R_1, R_2, V_S.

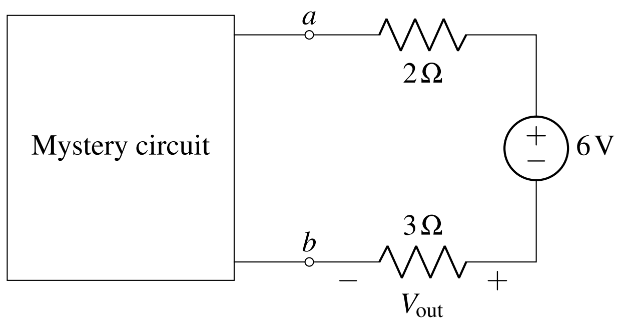




$I_{eq} =$

$R_{eq} =$

(b) (8 points) Consider a mystery circuit. You decide to connect a voltage source and two resistors to its terminals a and b as shown. You are told that the Thévenin equivalent circuit for the mystery circuit between nodes a and b has $V_{th} = 3\,\text{V}$ and $R_{th} = 1\,\Omega$.



i. What is the voltage V_{out} across the $3\,\Omega$ resistor?

$V_{out} =$ ☐ V

ii. What is the power P_{out} dissipated by the $3\,\Omega$ resistor?

$P_{\text{out}} =$ ☐ W

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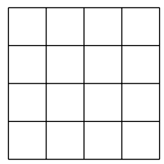
9. Noisy Imaging Revisited (12 points)

(a) (2 points) Oski decides to attend EECS 16A labs! He is given 4 different 2x2 masks for imaging:

Mask 1 Mask 2 Mask 3 Mask 4

What is the masking matrix that corresponds to the given masks? (Hint: remember that each row of a masking matrix represents a mask.)

Draw the masking matrix by filling in the squares of the grid below (draw an X in the squares that should be black/zero):



(b) (2 points) Could we use the masking matrix in the previous part to uniquely solve for an image from sensor data? **Remember that black squares represent zeros and white squares represent ones.**

◯ Yes ◯ No

Briefly justify your answer.

(c) (4 points) Recall that we modeled our noisy imaging system as $\vec{s} = H\vec{i} + \vec{\omega}$, where $\vec{s}$ is the measurements, H is the masking matrix, $\vec{i}$ is the true scene we want to reconstruct, and $\vec{\omega}$ is the noise vector. To reconstruct our image, we multiplied both sides of the equation by H^{-1}, yielding $\vec{i_{est}} = \vec{i} + H^{-1}\vec{\omega}$. The term $H^{-1}\vec{\omega}$ represents the effect of the noise on $\vec{i_{est}}$. In order to minimize $H^{-1}\vec{\omega}$, is it better for the eigenvalues of H to be small or large?

◯ Small ◯ Large

Justify your answer. You may assume that the eigenvectors of H span $\mathbb{R}^n$, where $\vec{\omega} \in \mathbb{R}^n$.

(d) (2 points) Another way to counteract the effect of noise is to collect more measurements and take advantage of redundancy using least squares, as you've learned in Module 3. In this case, our new masking matrix H would be tall (more rows than columns) instead of square, so we cannot invert it directly. Let's analyze the effect of noise in this scenario.

First, consider the case when there is no noise in the measurement: $\vec{\omega} = \vec{0}$ and $\vec{s} = H\vec{i}$. Using least squares, write a mathematical formula for the reconstructed image $\vec{i}_{clean}$ in terms of H and $\vec{s}$.

(e) (2 points) Next, consider the case when there is noise: $\vec{\omega} \neq \vec{0}$ and $\vec{s} = H\vec{i} + \vec{\omega}$. To see the effect of noise, use least squares again to write a mathematical formula for the reconstructed image $\vec{i}_{noisy}$ in terms of H, $\vec{\omega}$, and $\vec{i}_{clean}$ from the previous part.

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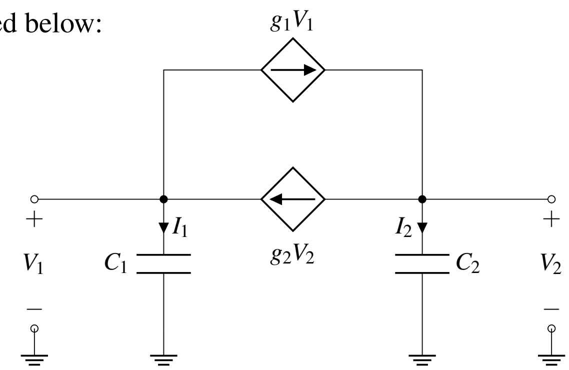
10. Charge Pumps (15 points)

Consider the circuit described below:



Two capacitors C_1 and C_2 are connected by two dependent current sources which control the flow of current between the capacitors. The charges on the capacitors can be modeled as discrete time signals $Q_1[t]$ and $Q_2[t]$ that are sampled every Δt seconds.

(a) (4 points) We are given that $Q_1[t], Q_2[t]$ change in time according to the following equations:

$$Q_1[t+1] = \left(1 - \frac{g_1 \Delta t}{C_1}\right) Q_1[t] + \left(\frac{g_2 \Delta t}{C_2}\right) Q_2[t]$$
$$Q_2[t+1] = \left(\frac{g_1 \Delta t}{C_1}\right) Q_1[t] + \left(1 - \frac{g_2 \Delta t}{C_2}\right) Q_2[t]$$

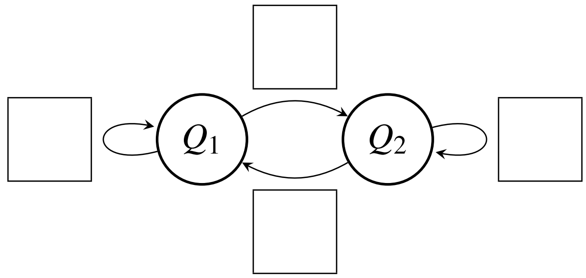
Let us define the vector

$$\vec{q}[t] = \begin{bmatrix} Q_1[t] \\ Q_2[t] \end{bmatrix}$$

that represents the charges on both capacitors at time step t.

i. Assuming $C_1 = 10\,\mu\text{F}$, $C_2 = 20\,\mu\text{F}$, $g_1 = 5\,\text{A}\cdot\text{V}^{-1}$, $g_2 = 4\,\text{A}\cdot\text{V}^{-1}$, $\Delta t = 1\,\mu\text{s}$, find the state transition matrix S where $\vec{q}[t+1] = S\vec{q}[t]$. Note: $1\,\text{F} = 1\,\text{A}\cdot\text{s}\cdot\text{V}^{-1}$.

ii. Fill in the state transition diagram that corresponds to S.



iii. Is this system conservative?

◯ Yes ◯ No

(b) (7 points) Let's assume we found the following state transition matrix for the charge pump circuit:

$$S = \begin{bmatrix} 1/2 & 1/4 \\ 1/2 & 3/4 \end{bmatrix}$$

Find the eigenvalues and corresponding eigenvectors of S.

$\lambda_1 =$ ☐ $\vec{v}_1 =$ ☐ $\lambda_2 =$ ☐ $\vec{v}_2 =$ ☐

(c) (4 points) For the next parts, assume that the charge pump state transition matrix is

$$S = \begin{bmatrix} 2/5 & 1/5 \\ 3/5 & 4/5 \end{bmatrix}$$

which has eigenvalue, eigenvector pairs:

$$\lambda_1 = 1, v_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \qquad \lambda_2 = \frac{1}{5}, v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

i. If we start in the state

$$\vec{q}[0] = \alpha \vec{v}_1 + \beta \vec{v}_2$$

where $\alpha \neq 0, \beta \neq 0$, does $\lim_{t\to\infty} \vec{q}[t]$ converge? Explain.

◯ Yes ◯ No

ii. We are given that the starting charges in the capacitors are $Q_1[0] = 10\,\mu\text{C}, Q_2[0] = 6\,\mu\text{C}$. What is $\lim_{t\to\infty} \vec{q}[t]$?

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11. Fun with EECS16(AB) (12 points)

(a) (5 points) Suppose that A, B, are in $\mathbb{R}^{n\times n}$. Prove that if AB is invertible, then BA is invertible.
Hint: For matrices A, B, in $\mathbb{R}^{n\times n}$, the following fact holds:

$$\det(A)\det(B) = \det(AB)$$

(b) (7 points) Prove the following theorem: If $A \in \mathbb{R}^{n\times m}$ and $B \in \mathbb{R}^{m\times n}$ for $n > m$, then AB is not invertible. In other words, if A is a tall matrix and B is a wide matrix, then their product AB is not invertible.

Hint 1: What must be true about the nullspace of a matrix if it is not invertible?

Hint 2: What must be true about the nullspace of a wide matrix?

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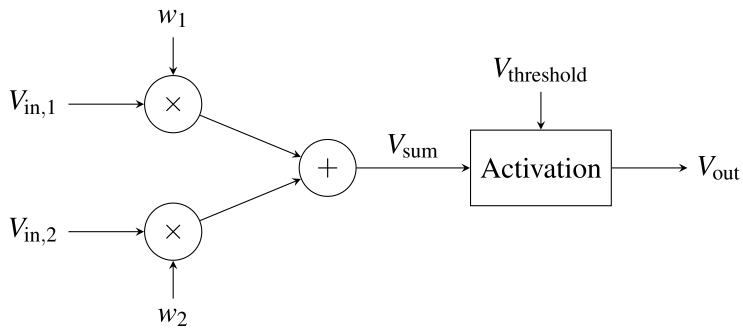
12. Artificial Neuron (13 points)

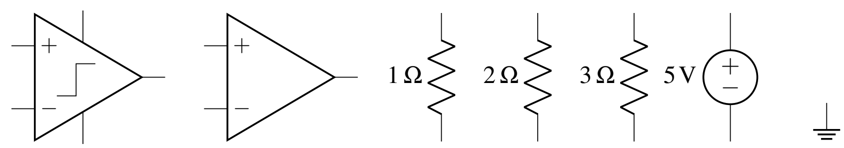
EECS16A is trying to build an AI that can teach the course without any professors or TAs. You are tasked with designing the artificial neuron circuit below that will be used as a core component of the EECS16A AI neural network:

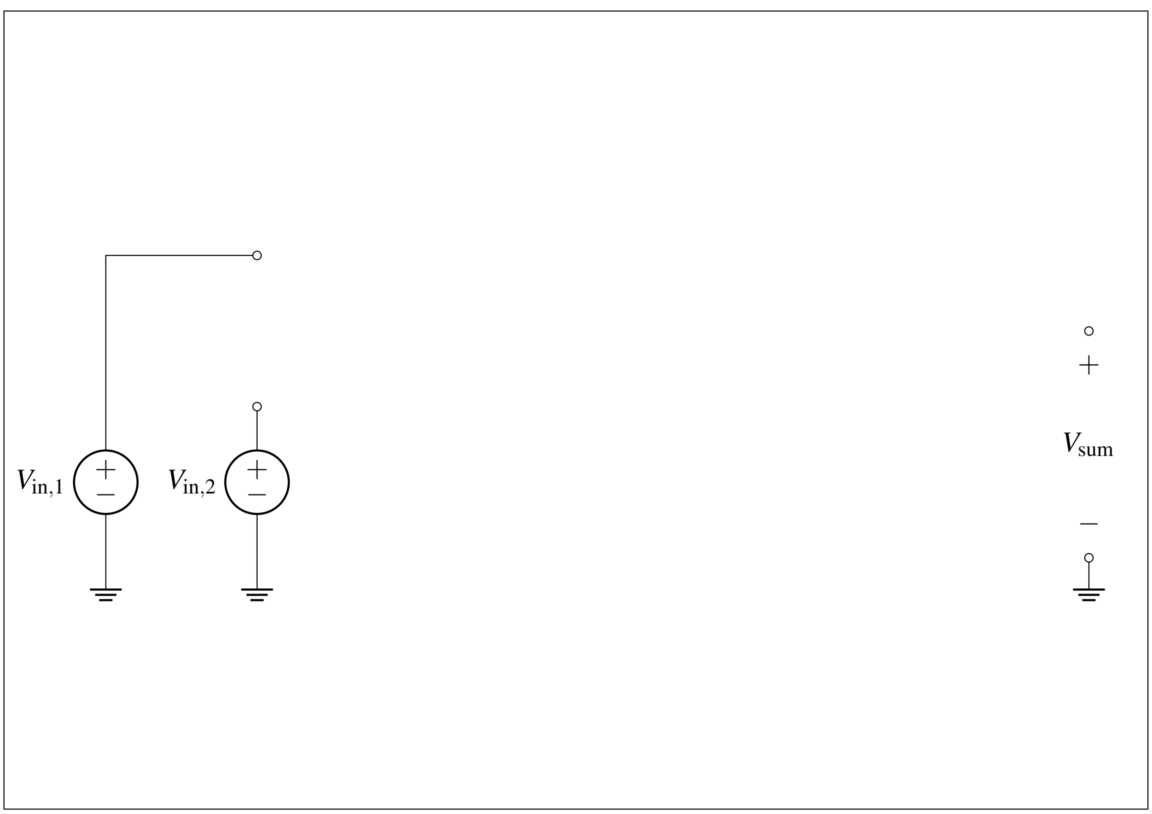


(a) (8 points) Let's design the first part of the neuron which is a weighted summer $V_{\text{sum}} = w_1 V_{\text{in},1} + w_2 V_{\text{in},2}$. For our neuron, we are given **negative weights** $w_1 = -1.5, w_2 = -1$. Design a circuit that implements

$$V_{\text{sum}} = -1.5 V_{\text{in},1} - V_{\text{in},2}$$

using the components below. **You may use multiple of each component. You may not need to use every component.**

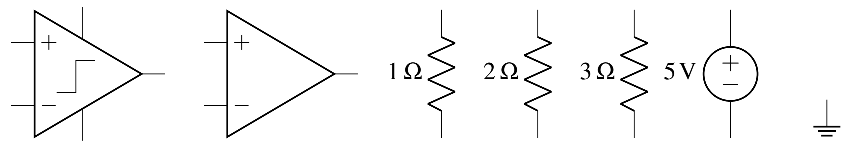




(b) (4 points) We now need to design the second portion of the circuit. Just like an actual neuron, our artificial neuron should only fire if $V_{\text{sum}} > V_{\text{threshold}}$. When the artificial neuron is inactive, we want $V_{\text{out}} = 0\,\text{V}$. When the artificial neuron fires, we want $V_{\text{out}} = 5\,\text{V}$. For our neuron, we are given that $V_{\text{threshold}} = 2.5\,\text{V}$. Design a circuit that implements

$$V_{\text{out}} = \begin{cases} 0\,\text{V} & V_{\text{sum}} < 2.5\,\text{V} \\ 5\,\text{V} & V_{\text{sum}} > 2.5\,\text{V} \end{cases}$$

using the components below. **You may use multiple of each component. You may not need to use every component.**



(c) (1 point) When we connect two circuits to one another, we need to be careful about the effects of loading. For the two parts of the artificial neuron circuit we designed, can we directly connect the output of the weighted summer to the input of the thresholding circuit or do we need to add a buffer in between?

◯ We need a buffer between the two circuits.

◯ We *do not* need a buffer between the two circuits.

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